From Turbulence to Reconnection to Particle Acceleration: Connecting the Dots

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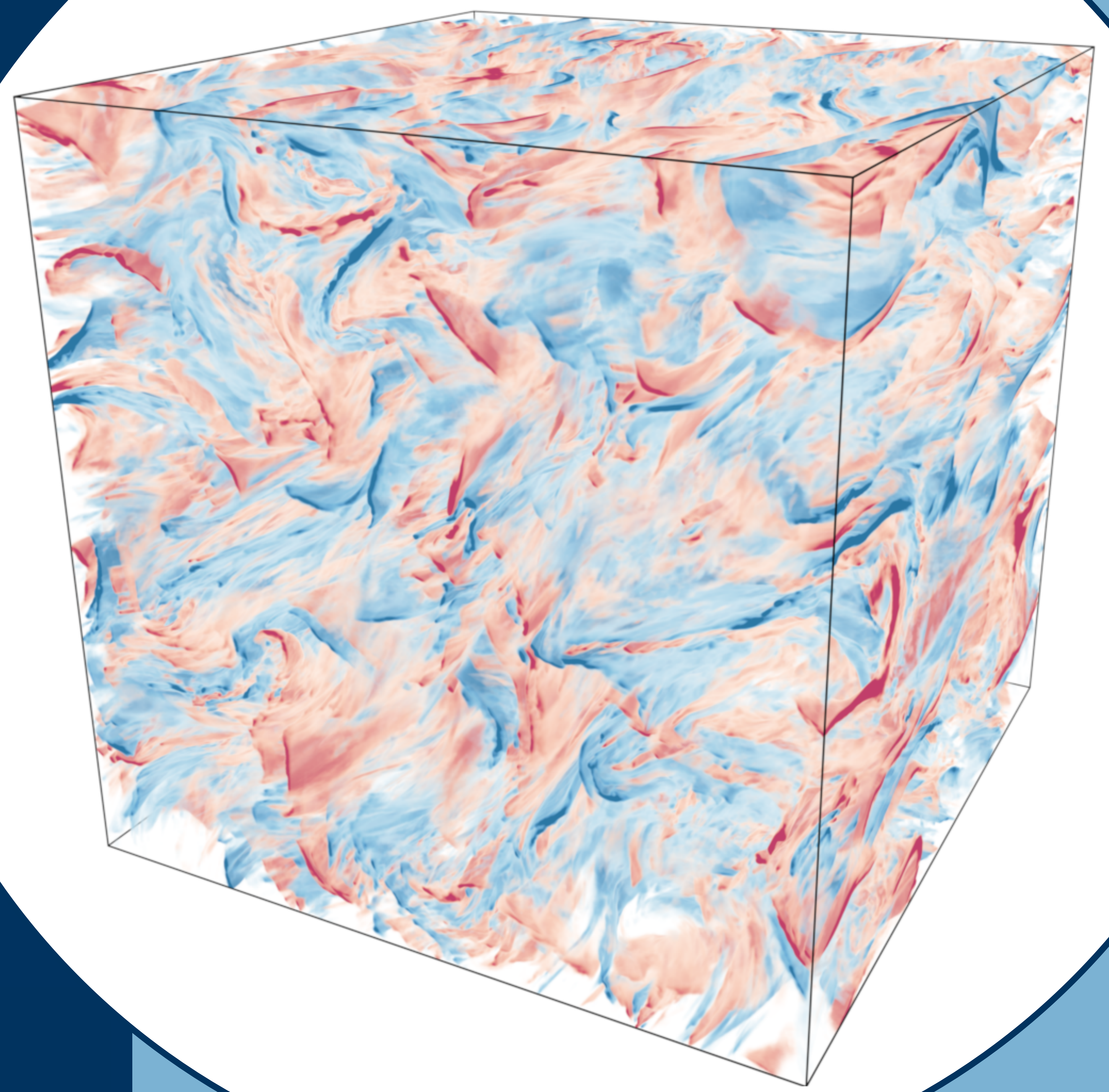
CDY Seminar - May 17, 2023



COLUMBIA UNIVERSITY
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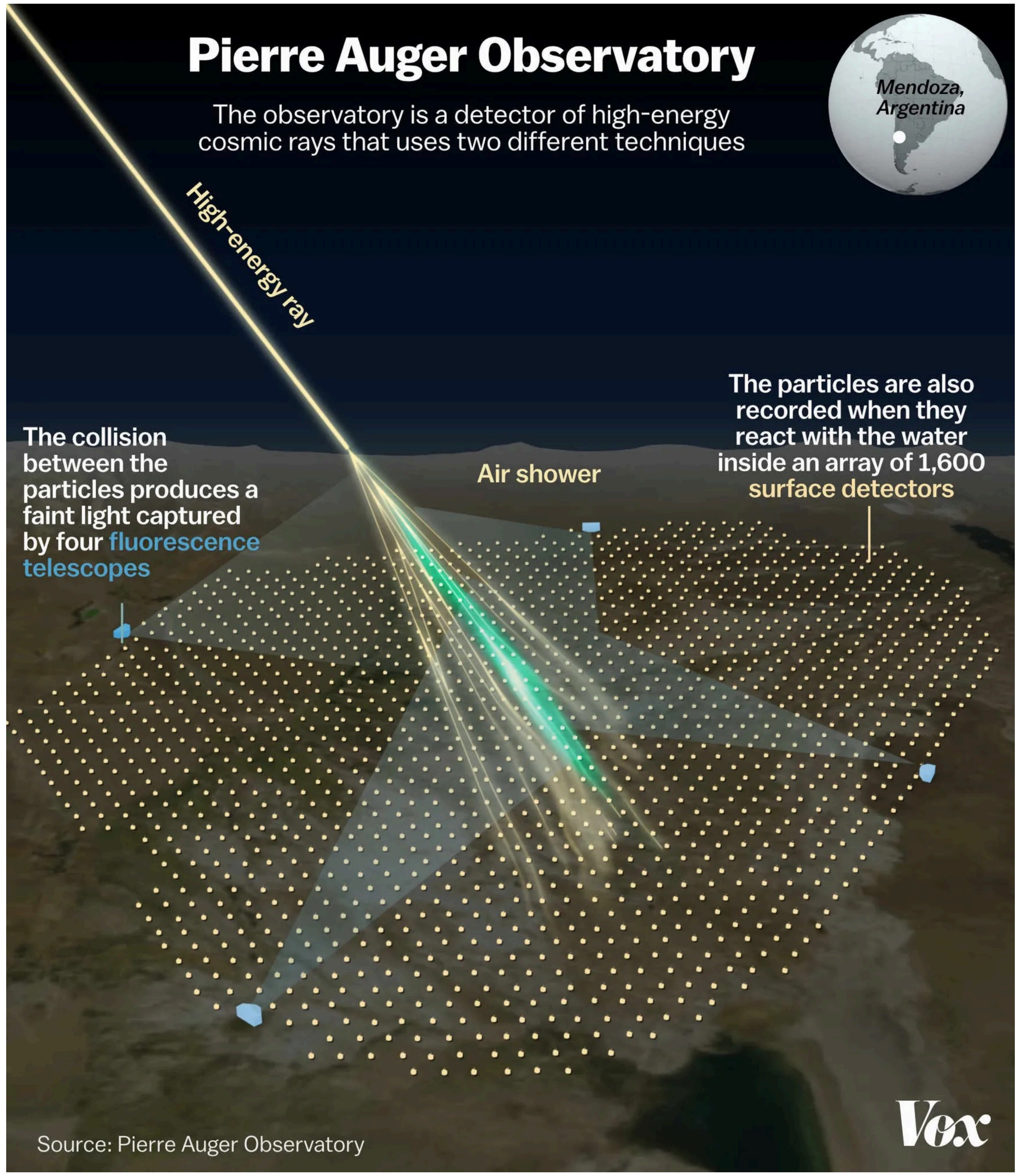
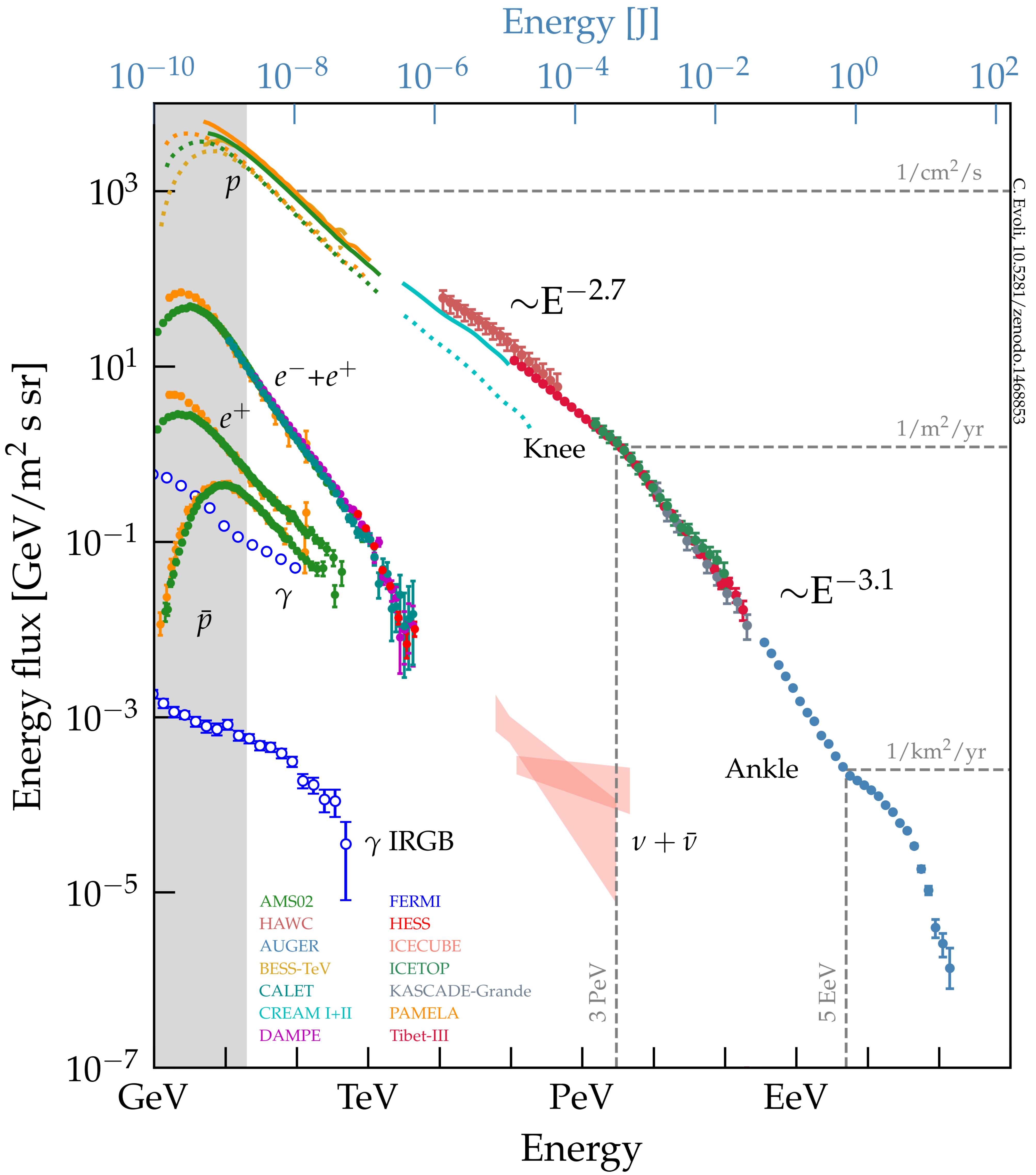
Plan of the talk

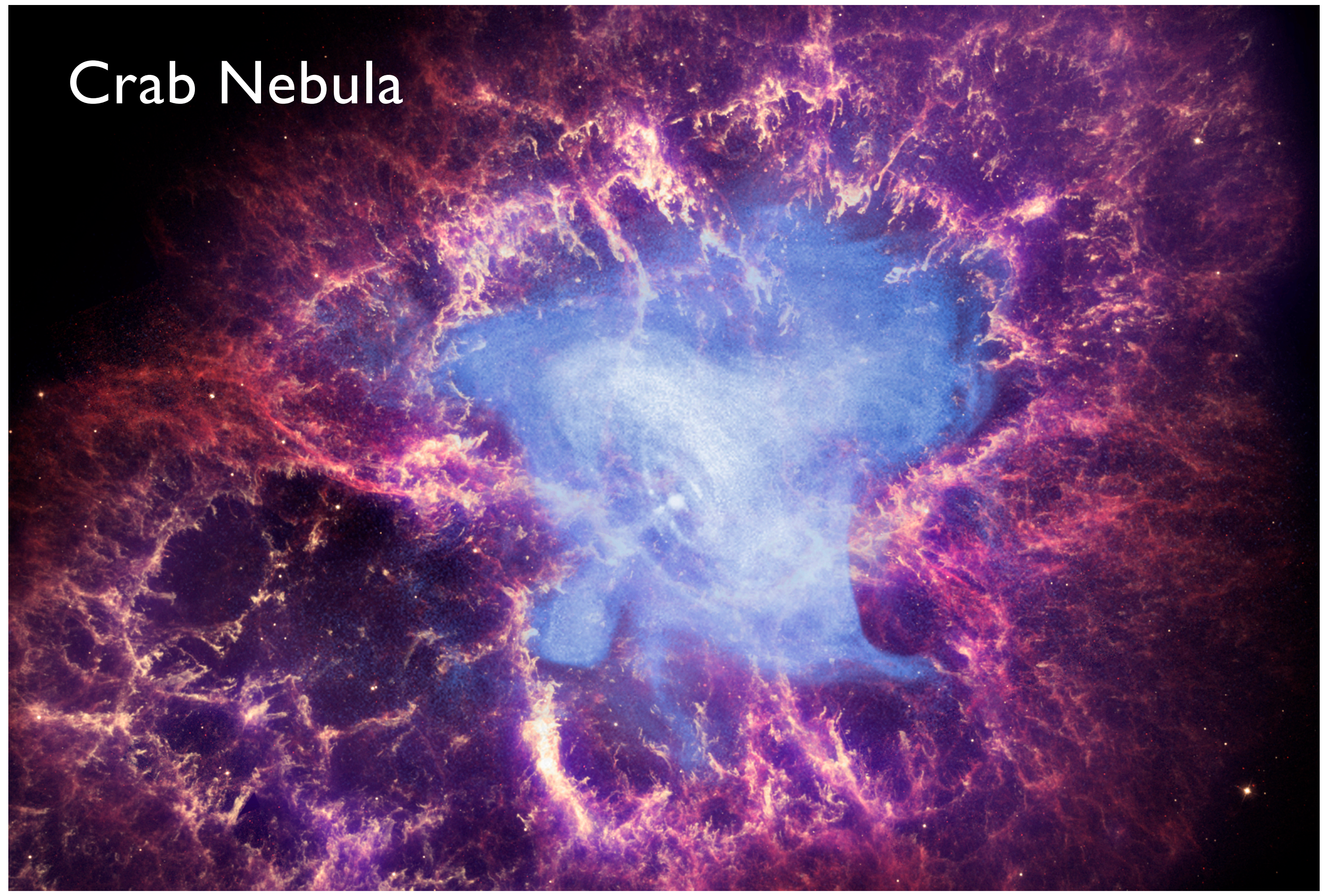
- The Non-Thermal Universe: Outstanding Examples
- Turbulence in Highly Magnetized Plasmas
- Relativistic Turbulence - high-σ regime
  - Pulsar Wind Nebulae
  - Gamma-Ray Bursts
- Non-Relativistic Turbulence - low-β regime
  - Solar Corona
- A few key takeaways

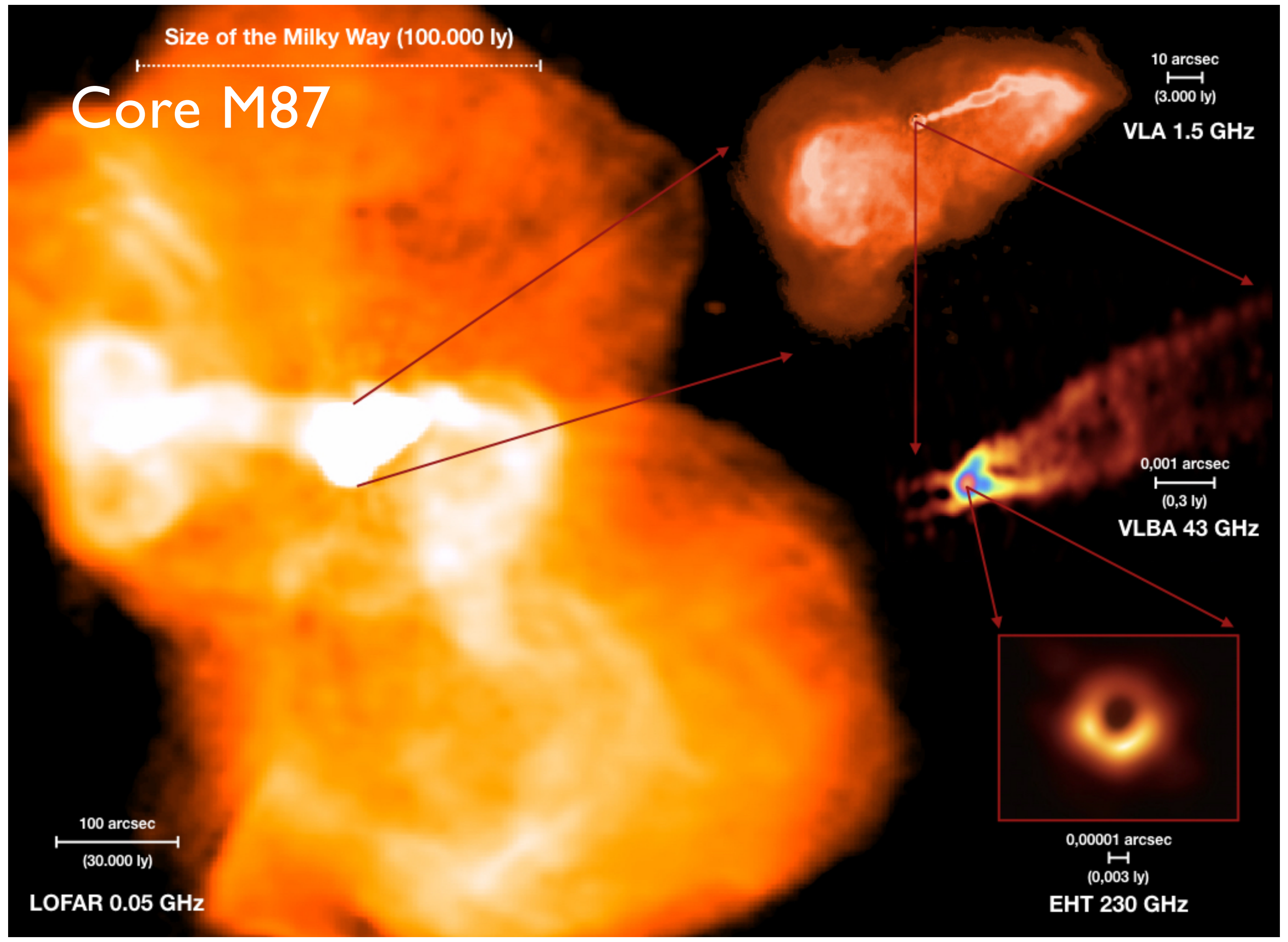
Energy spectrum of cosmic rays



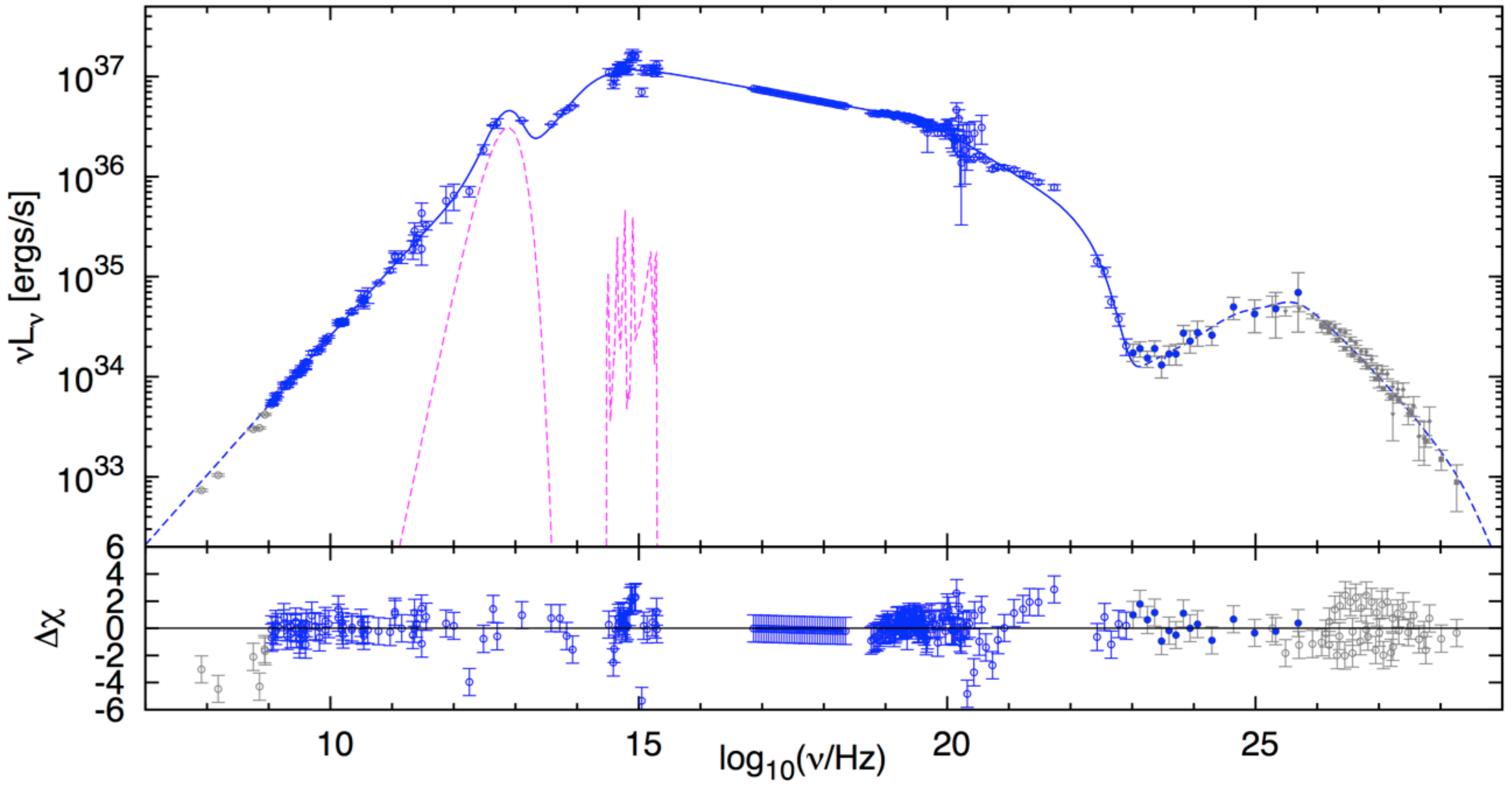


Sources of non-thermal radiation

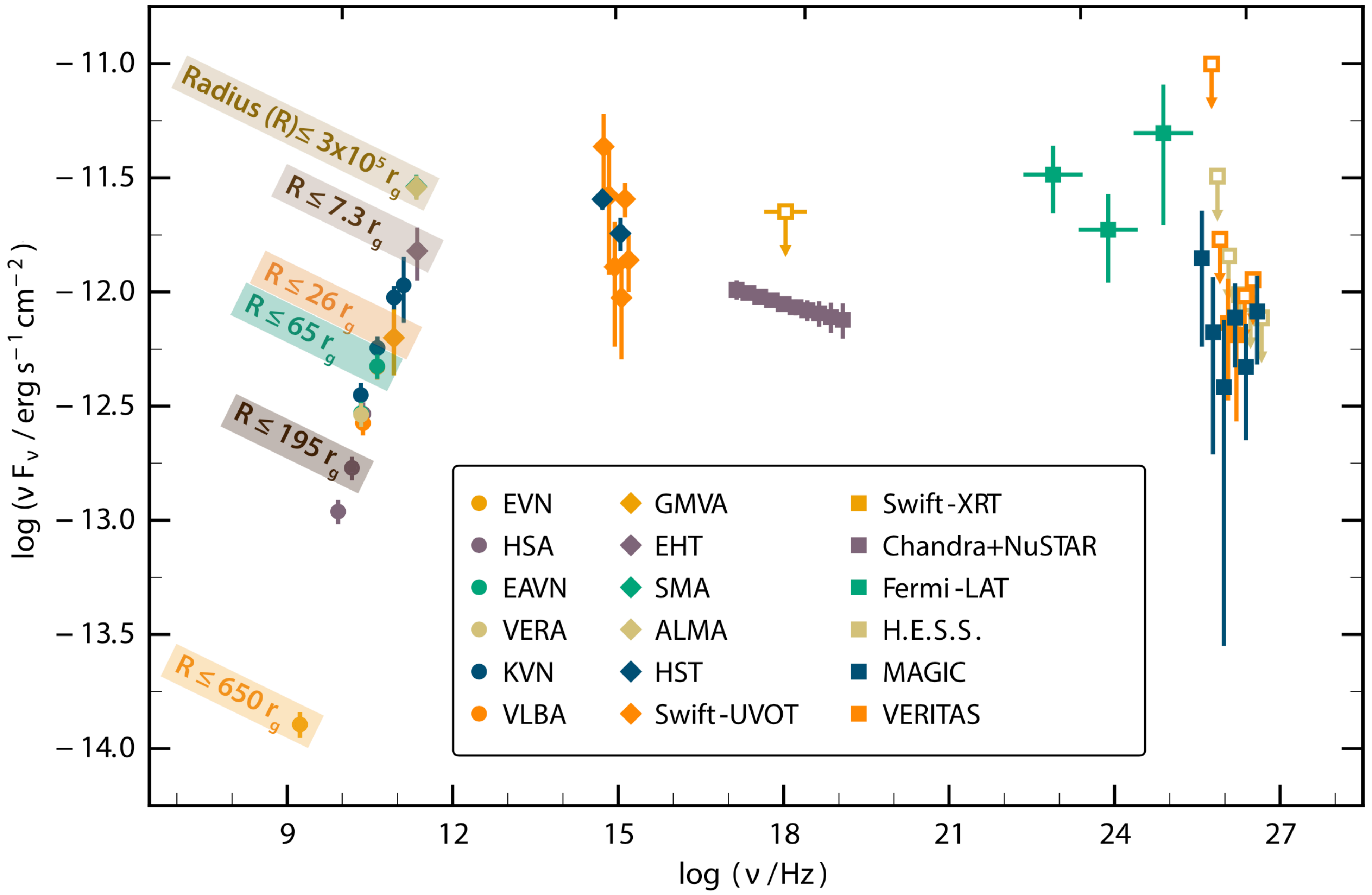




Crab Nebulae (Meyer et al. 2010)

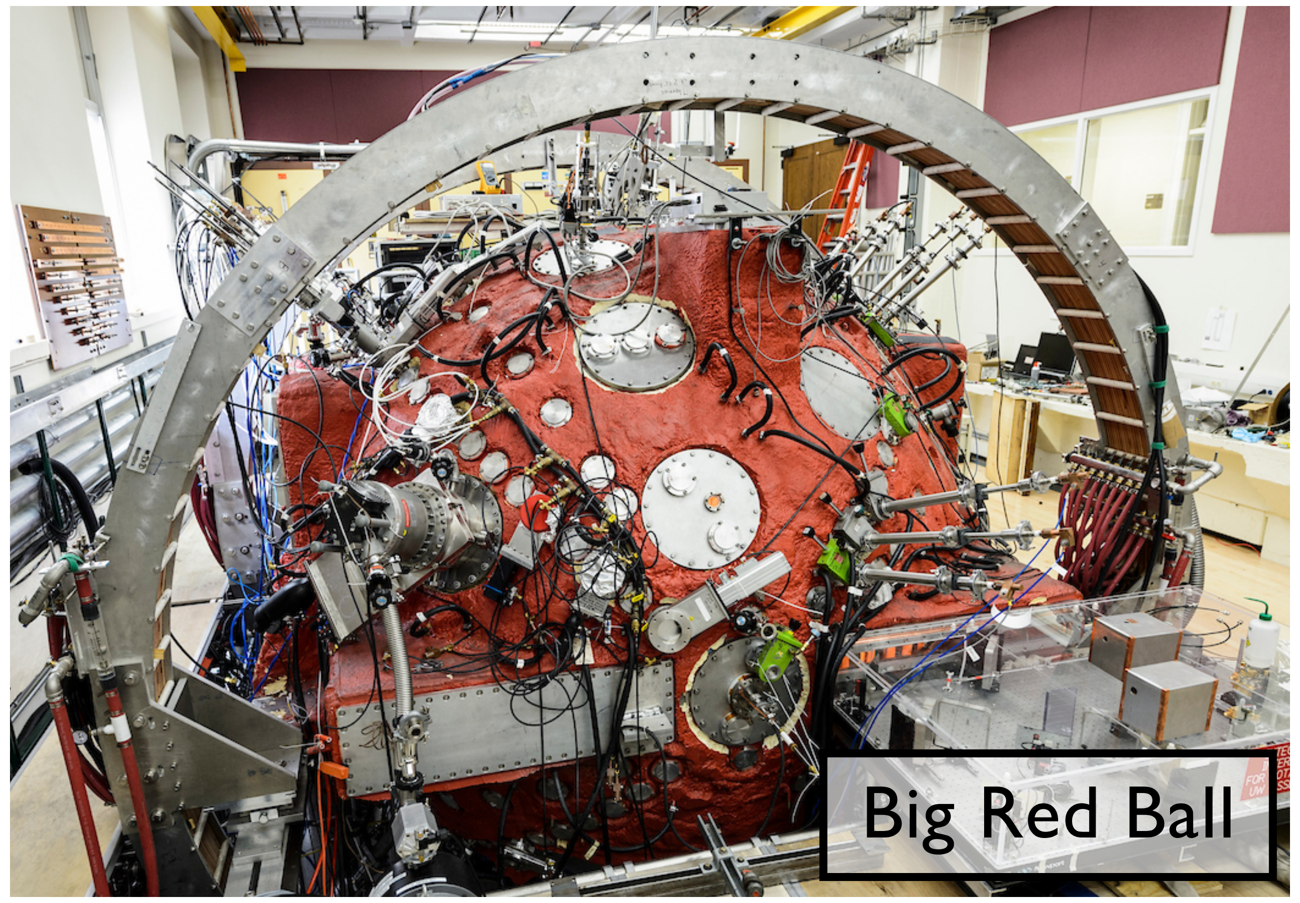


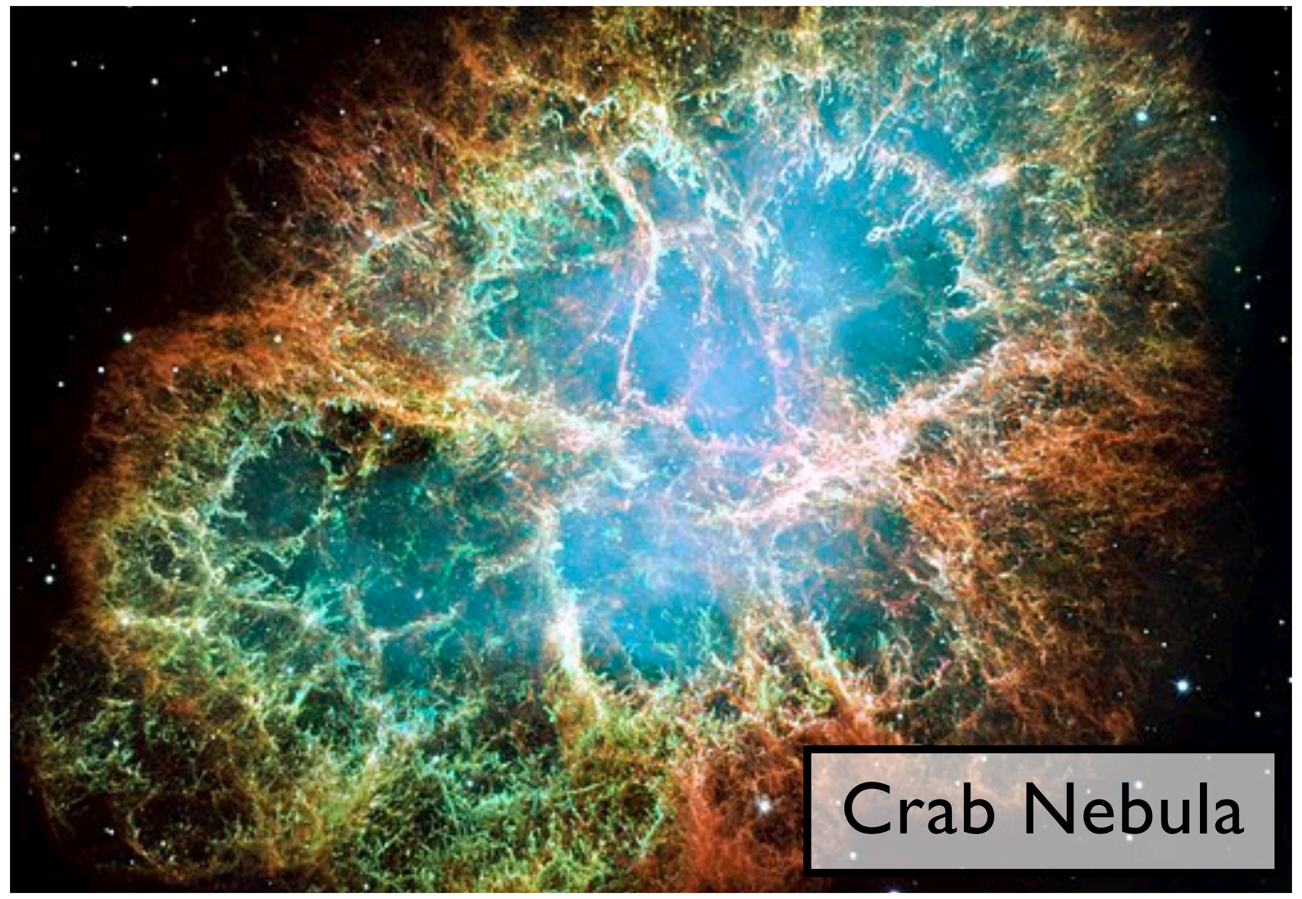
Core M87 Galaxy (Algaba et al. 2021)

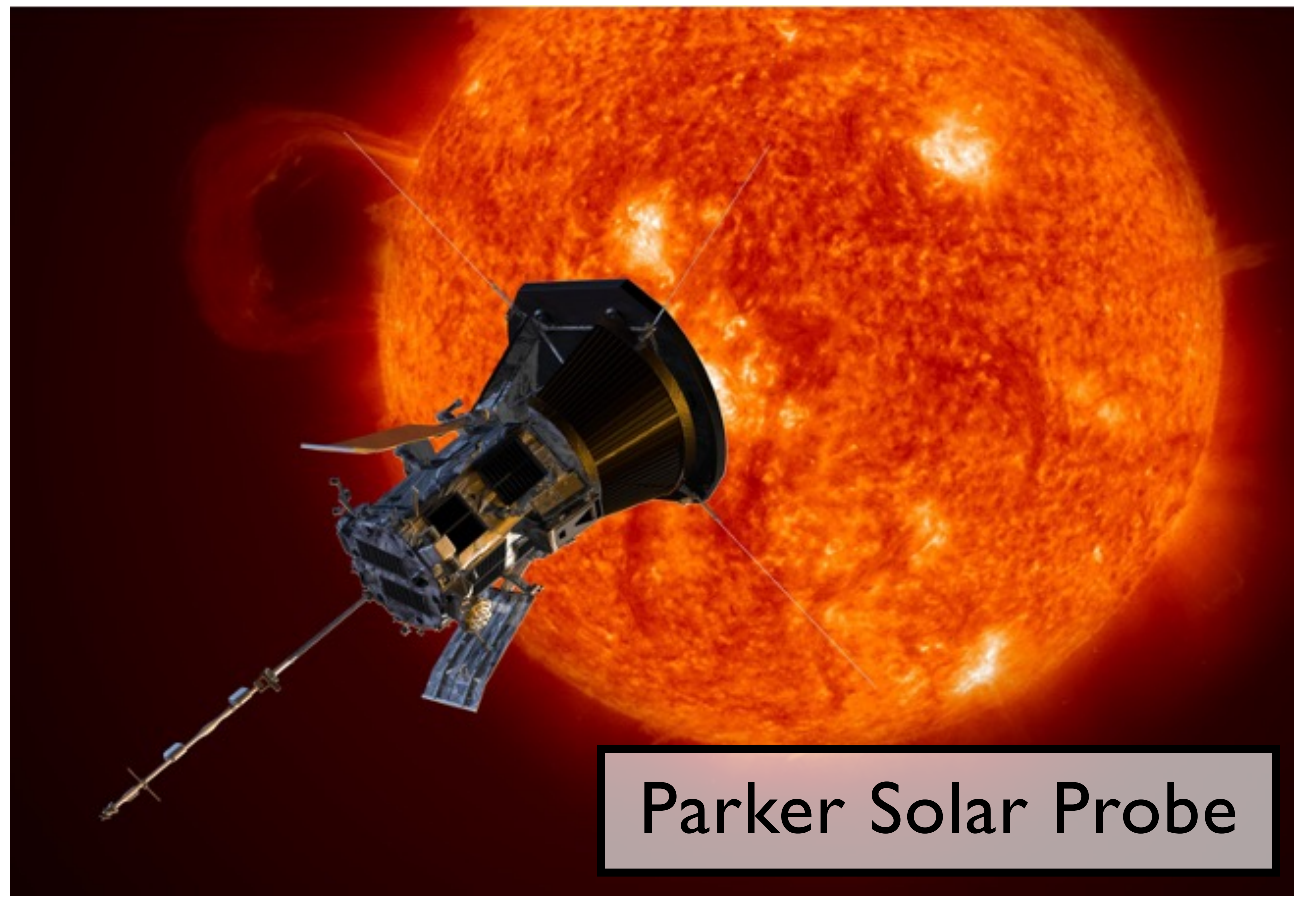


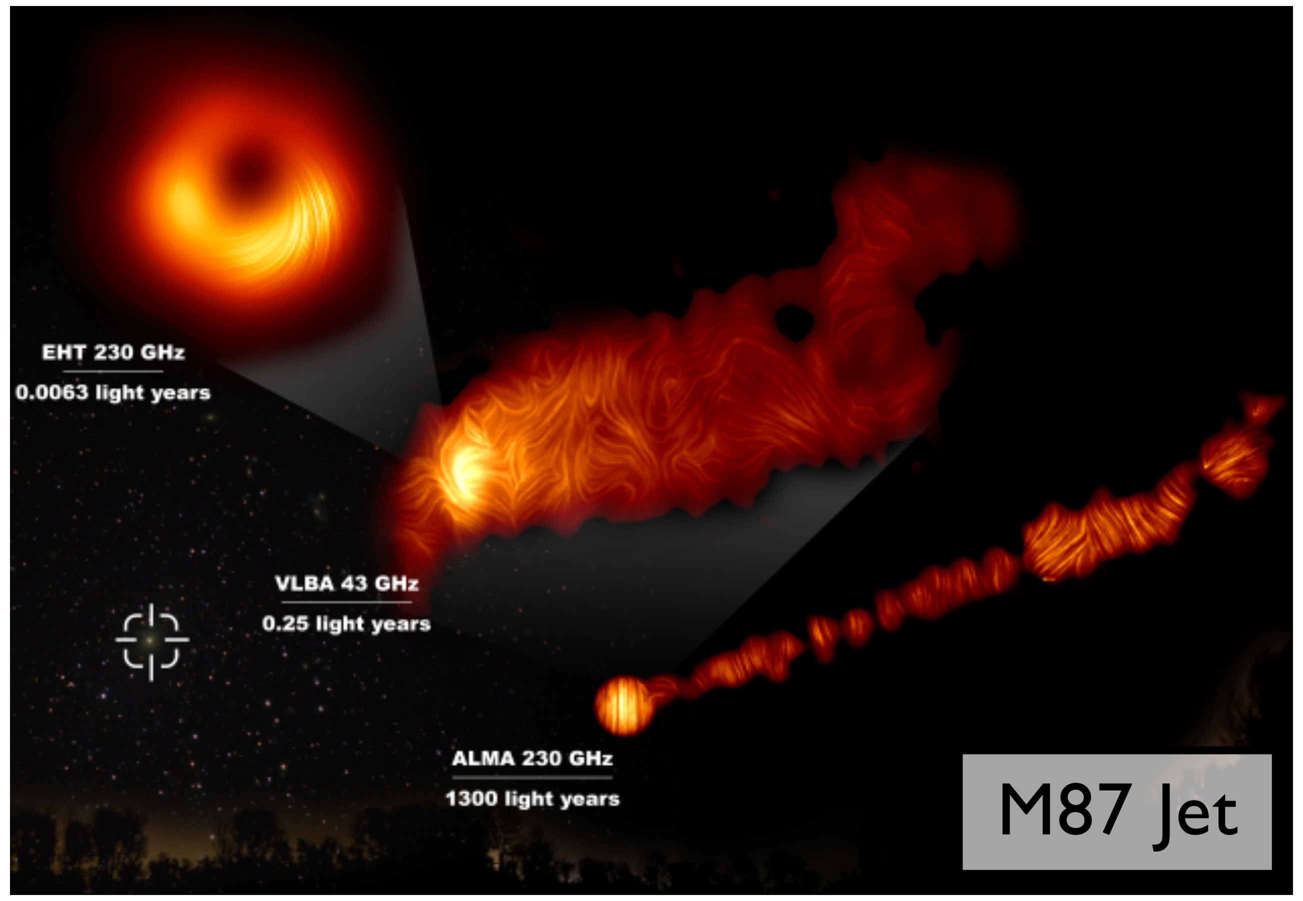
Magnetized environments (nonrelativistic vs relativistic regimes)

$\sigma \ll 1$ $\qquad \sigma = \dfrac{B^2}{4\pi\rho c^2} \simeq \dfrac{4}{\beta}\dfrac{k_B T}{m_i c^2}$ $\qquad \sigma \gg 1$

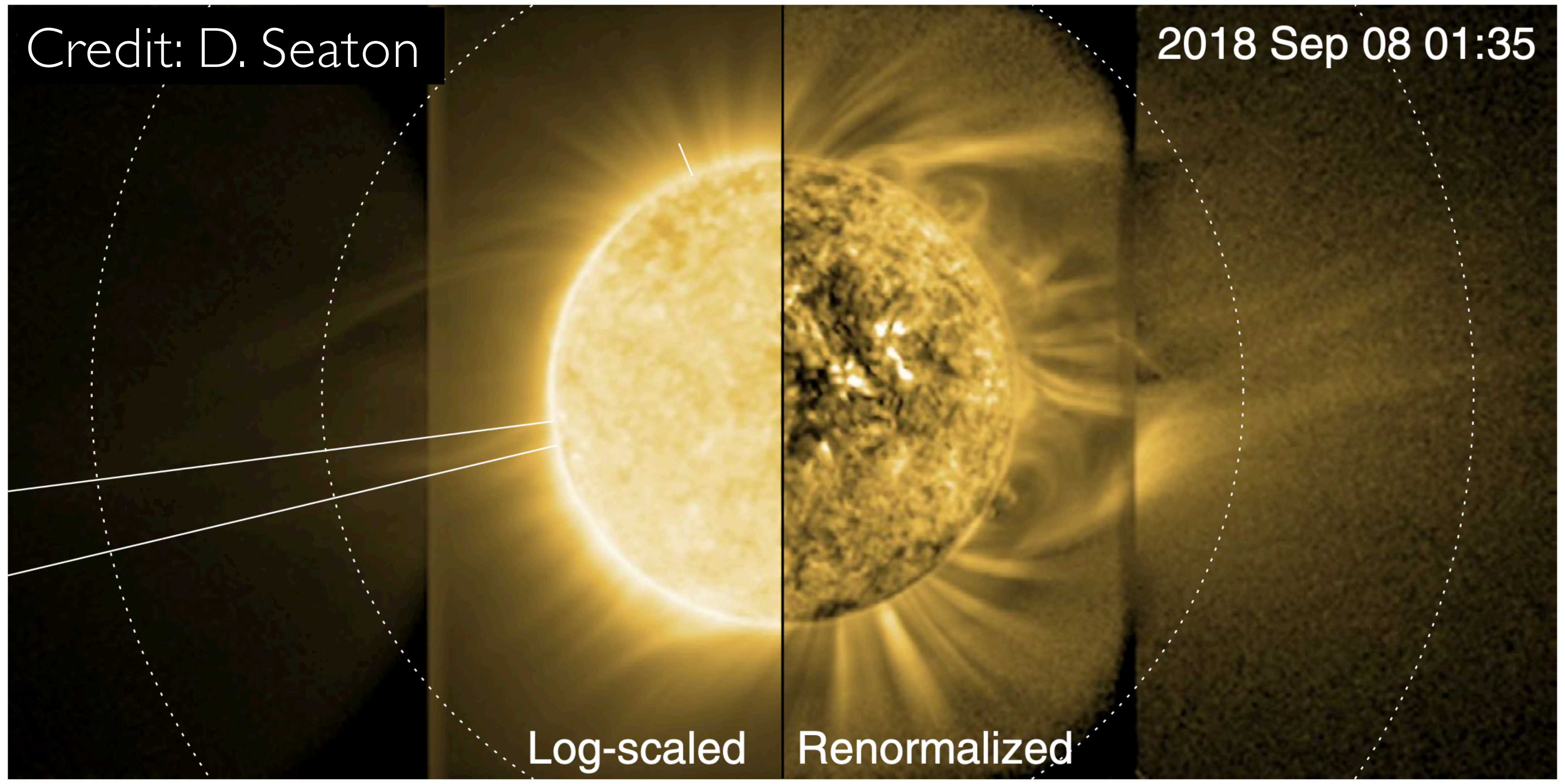


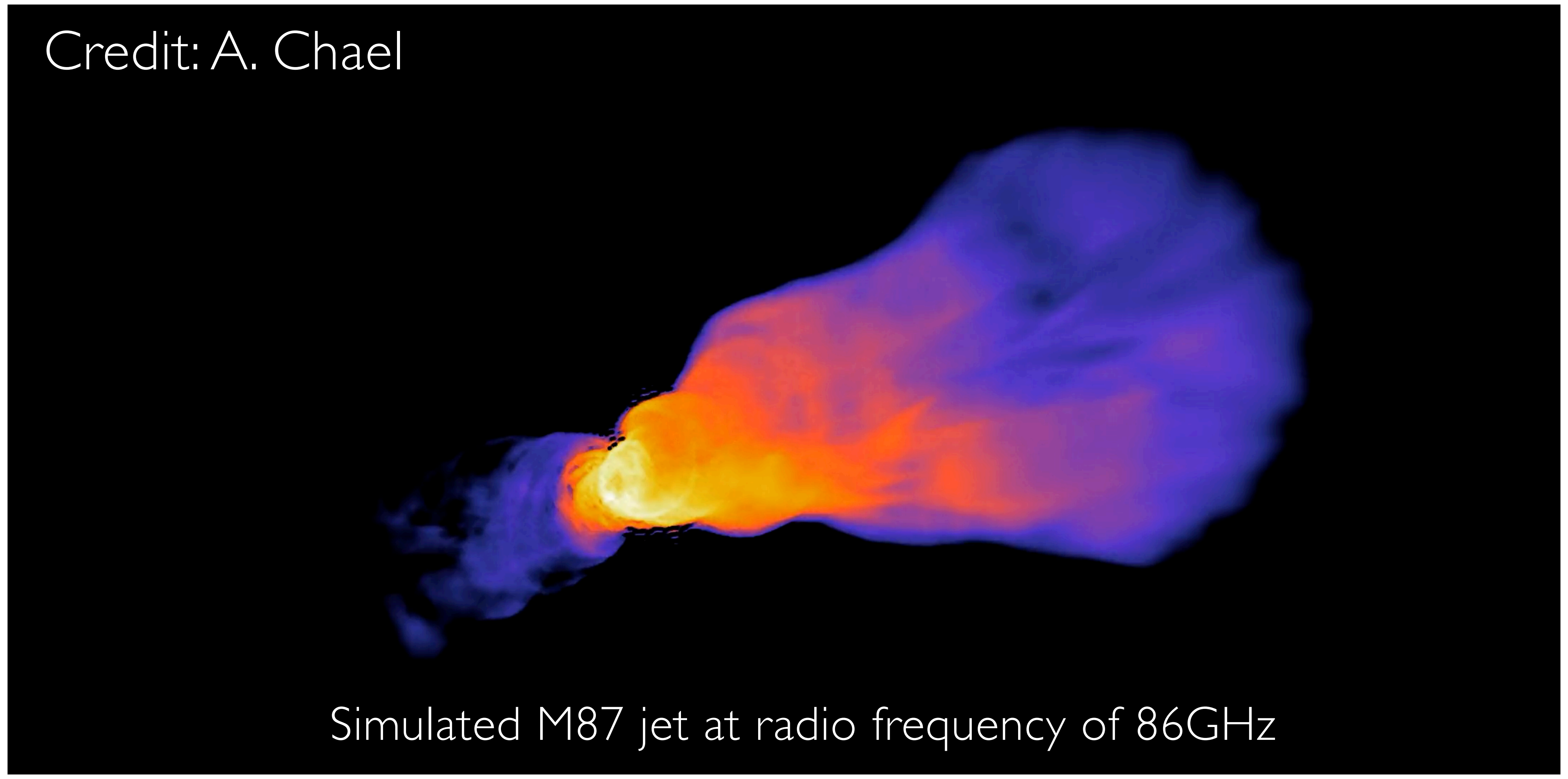






Large scale separation





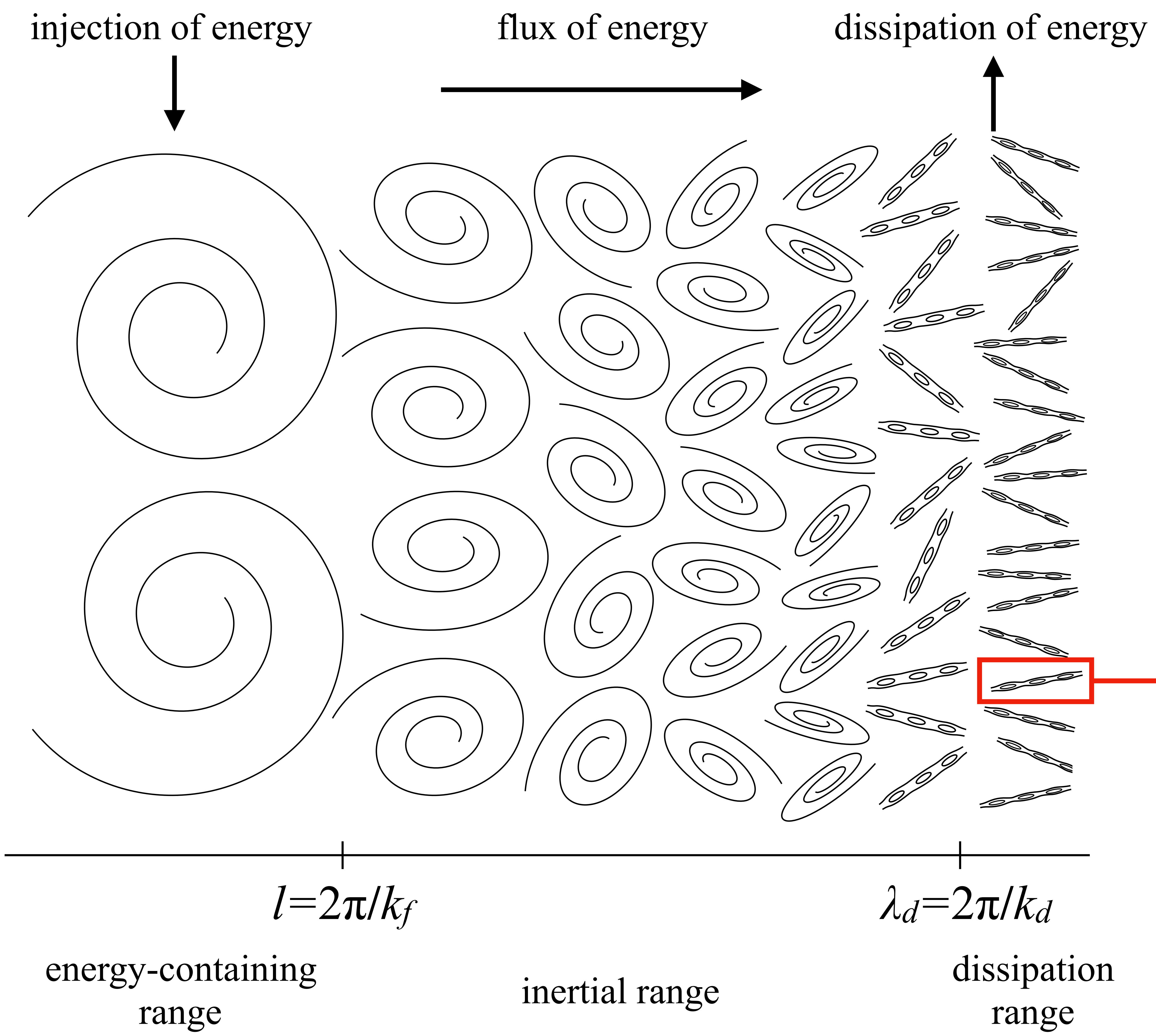
- Upper Solar Corona ($\sim 10^8\,\mathrm{m}$ above photosphere):

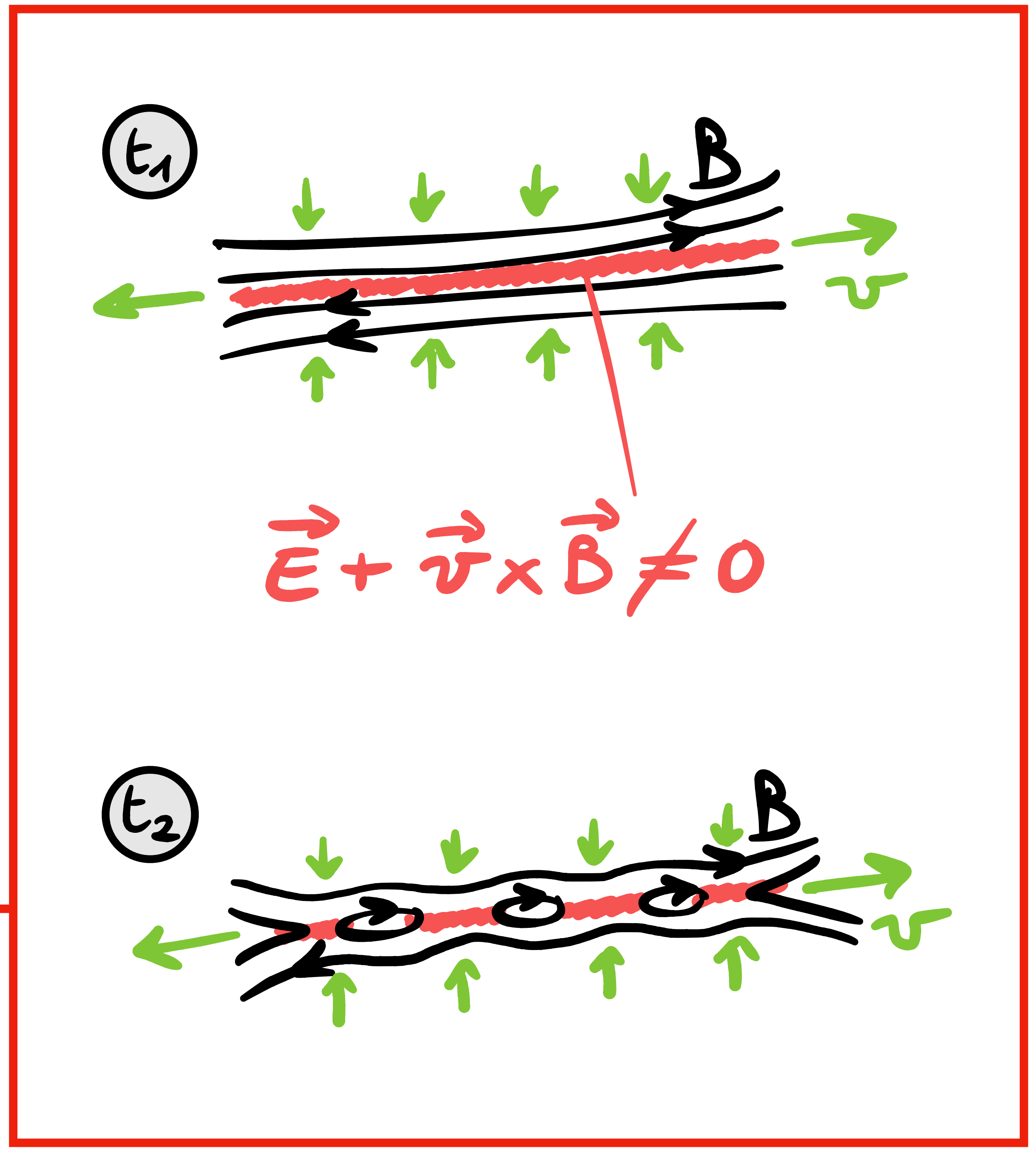
$$\lambda_{\mathrm{mfp},p} \sim 10^8\,\mathrm{m}$$
$$\ell_{\mathrm{kin}} \sim \rho_p \sim 10\,\mathrm{m}$$
$$(R_\odot \sim 7 \times 10^8\,\mathrm{m})$$

- M87* (close to event horizon, with large uncertainties):

$$\lambda_{\mathrm{mfp},p} \sim 10^{20}\,\mathrm{m}$$
$$\ell_{\mathrm{kin}} \sim \rho_p \sim 5 \times 10^3\,\mathrm{m}$$
$$(R_S \sim 2 \times 10^{12}\,\mathrm{m})$$

Turbulent energy cascade in large magnetized systems

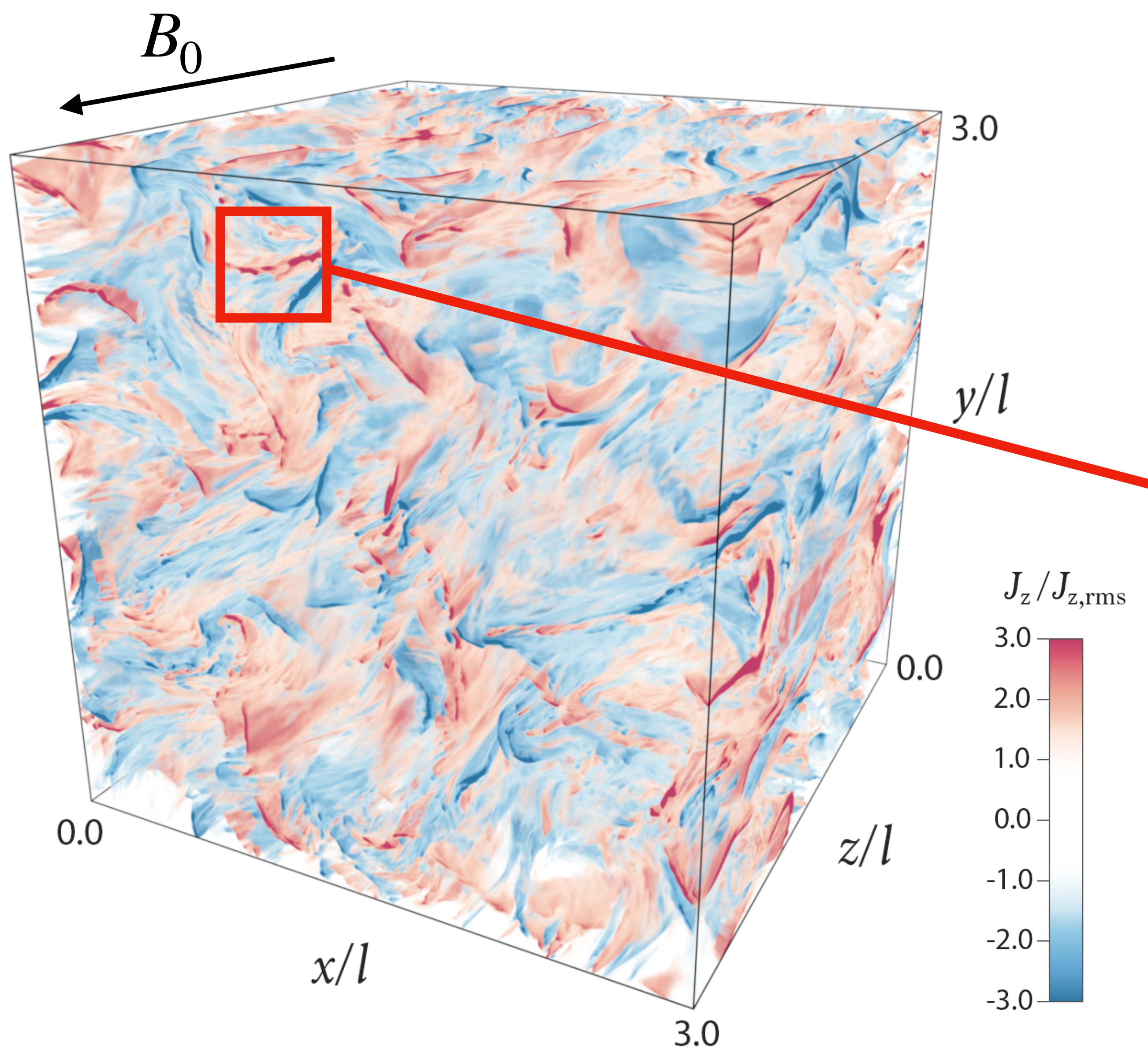


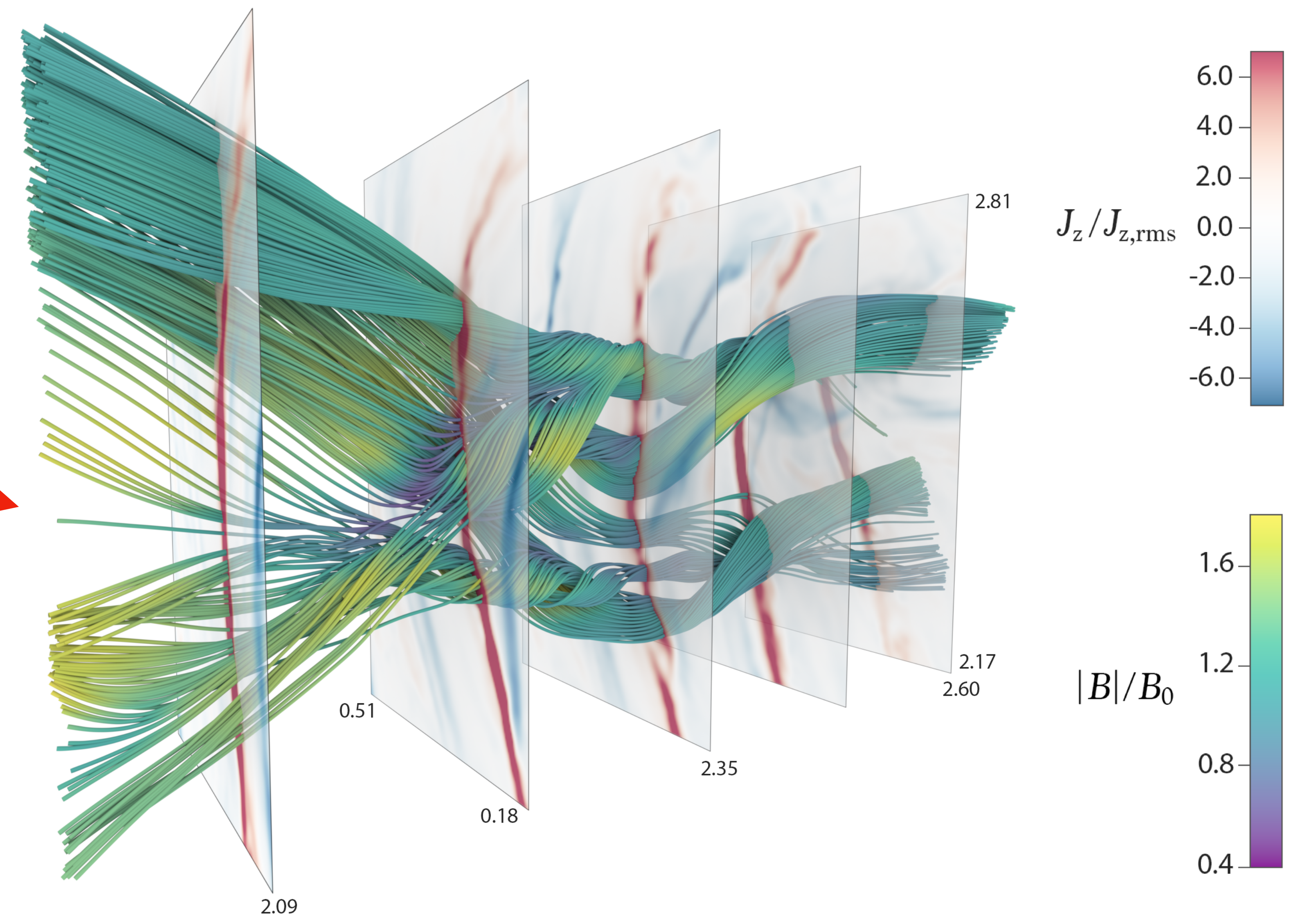


Turbulent energy cascade in large magnetized systems

Particle-in-Cell

Comisso and Sironi 2022

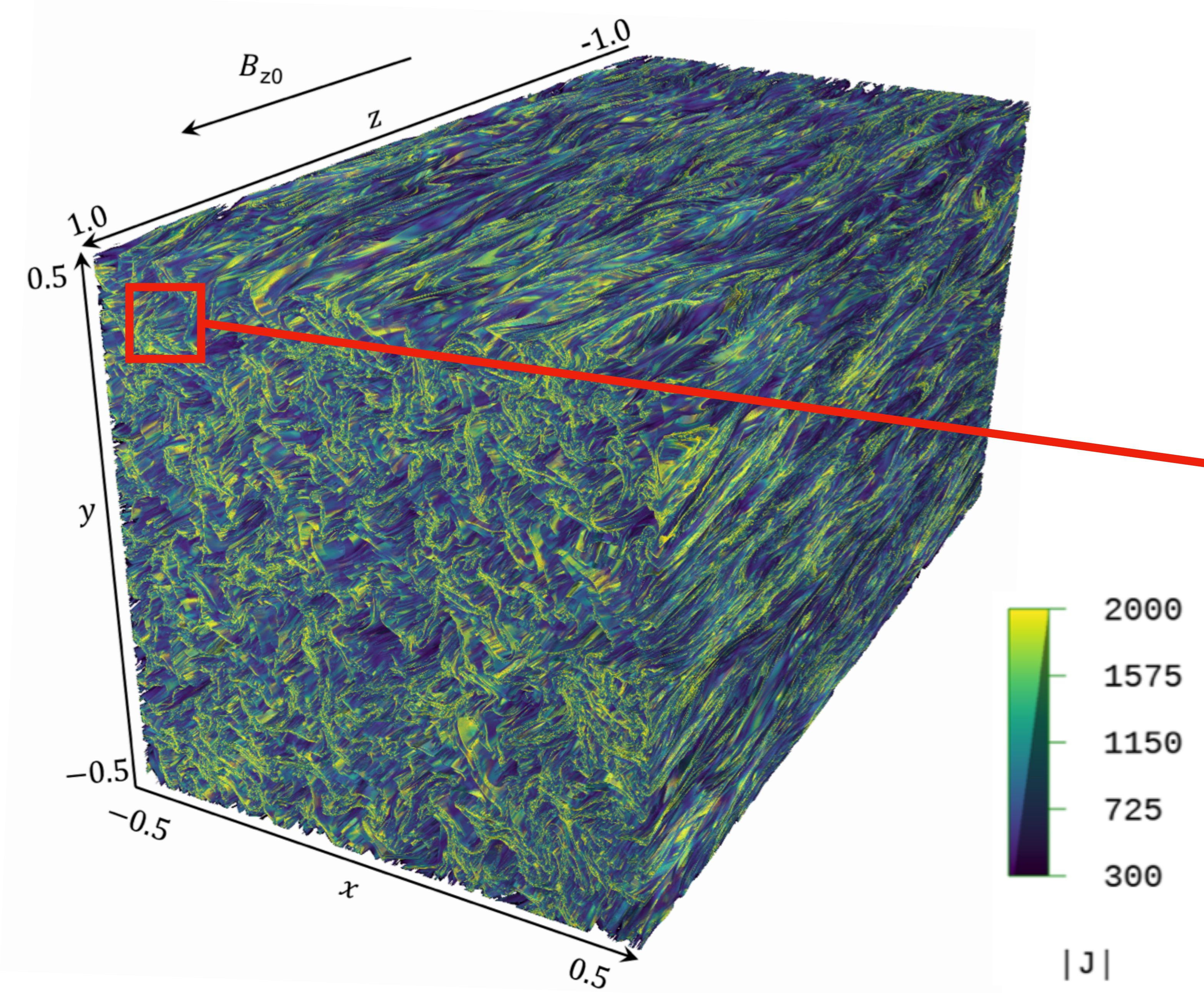


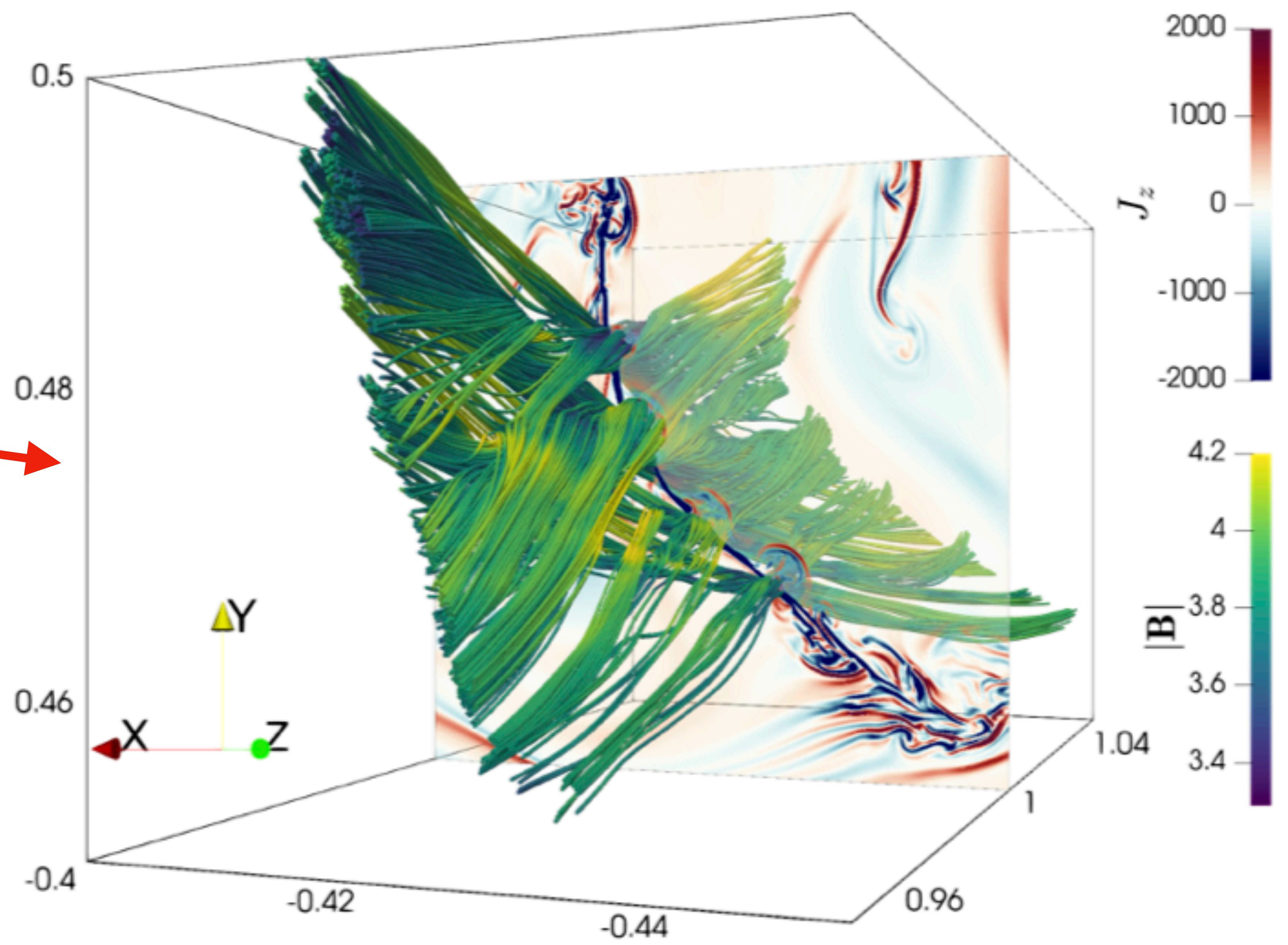


Magnetic Reconnection

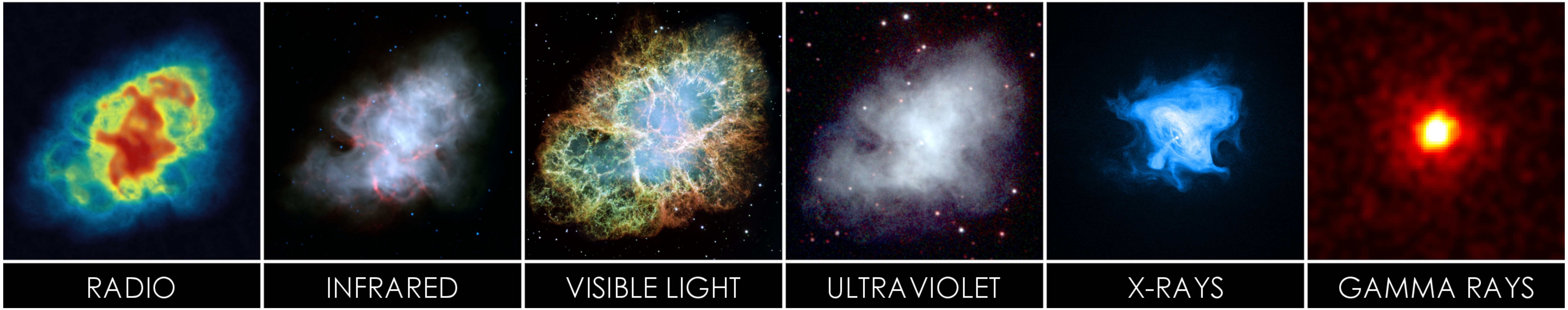
MHD

Dong+ 2022



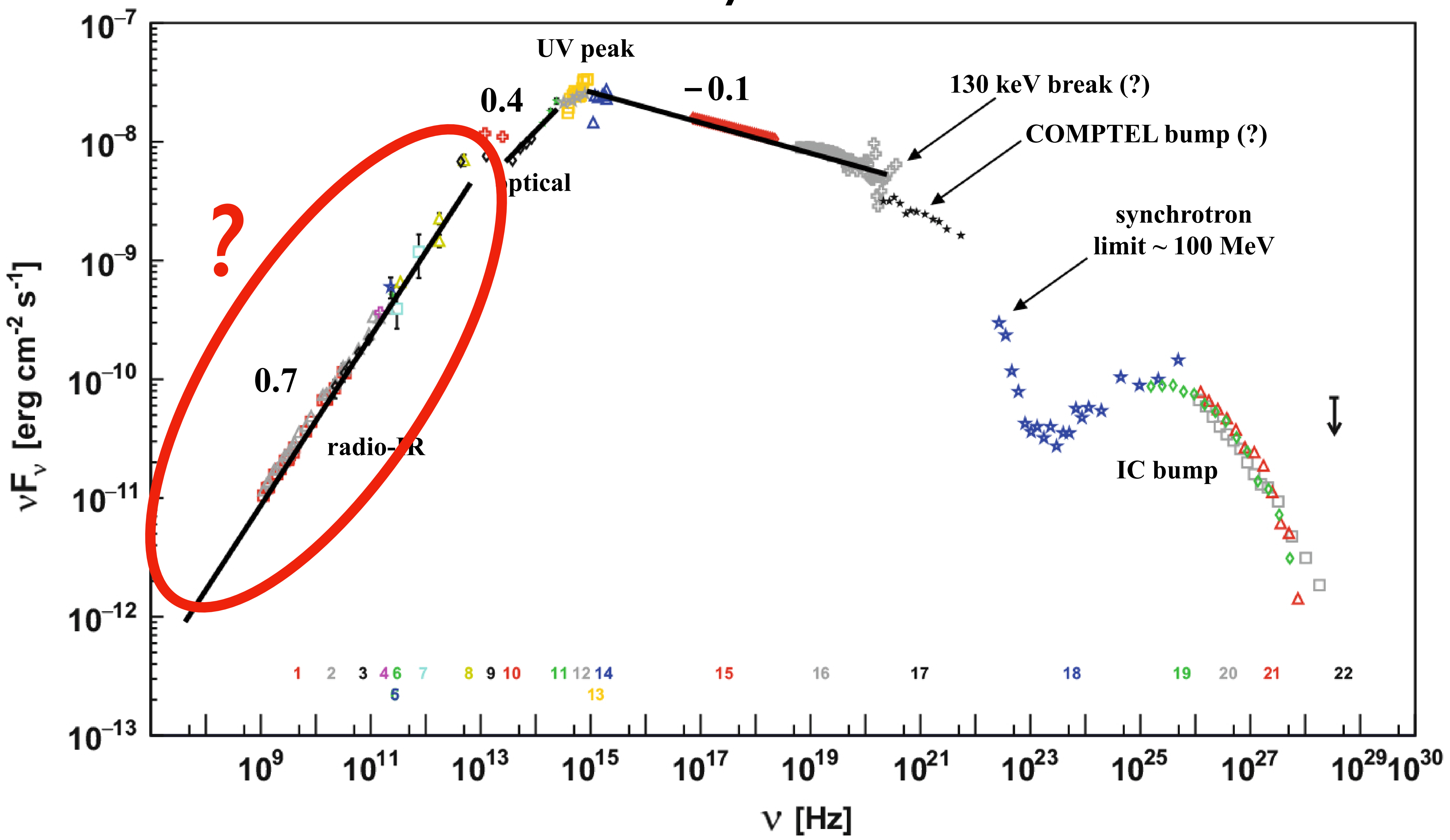


Relativistic turbulence in pulsar wind nebulae

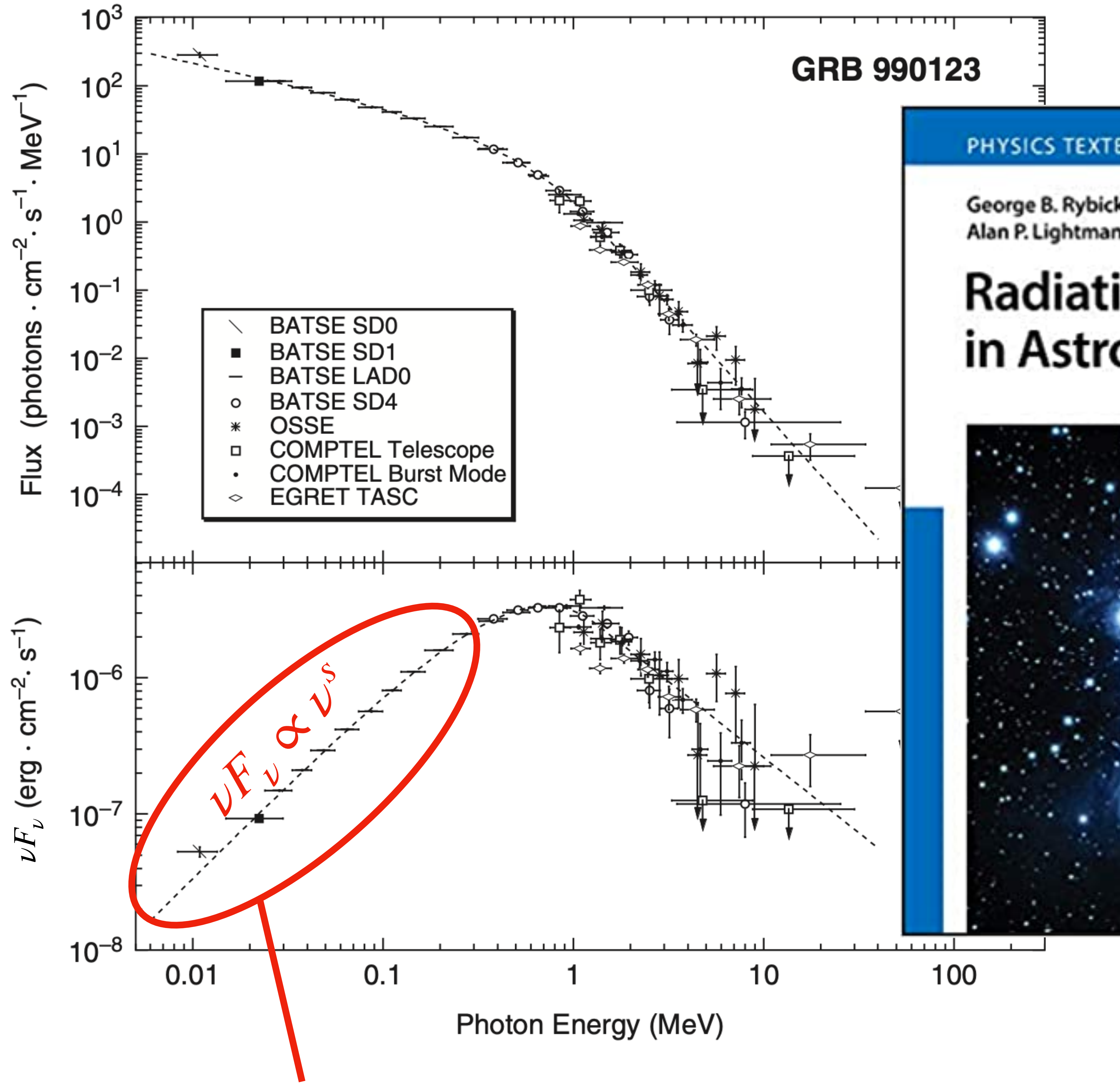


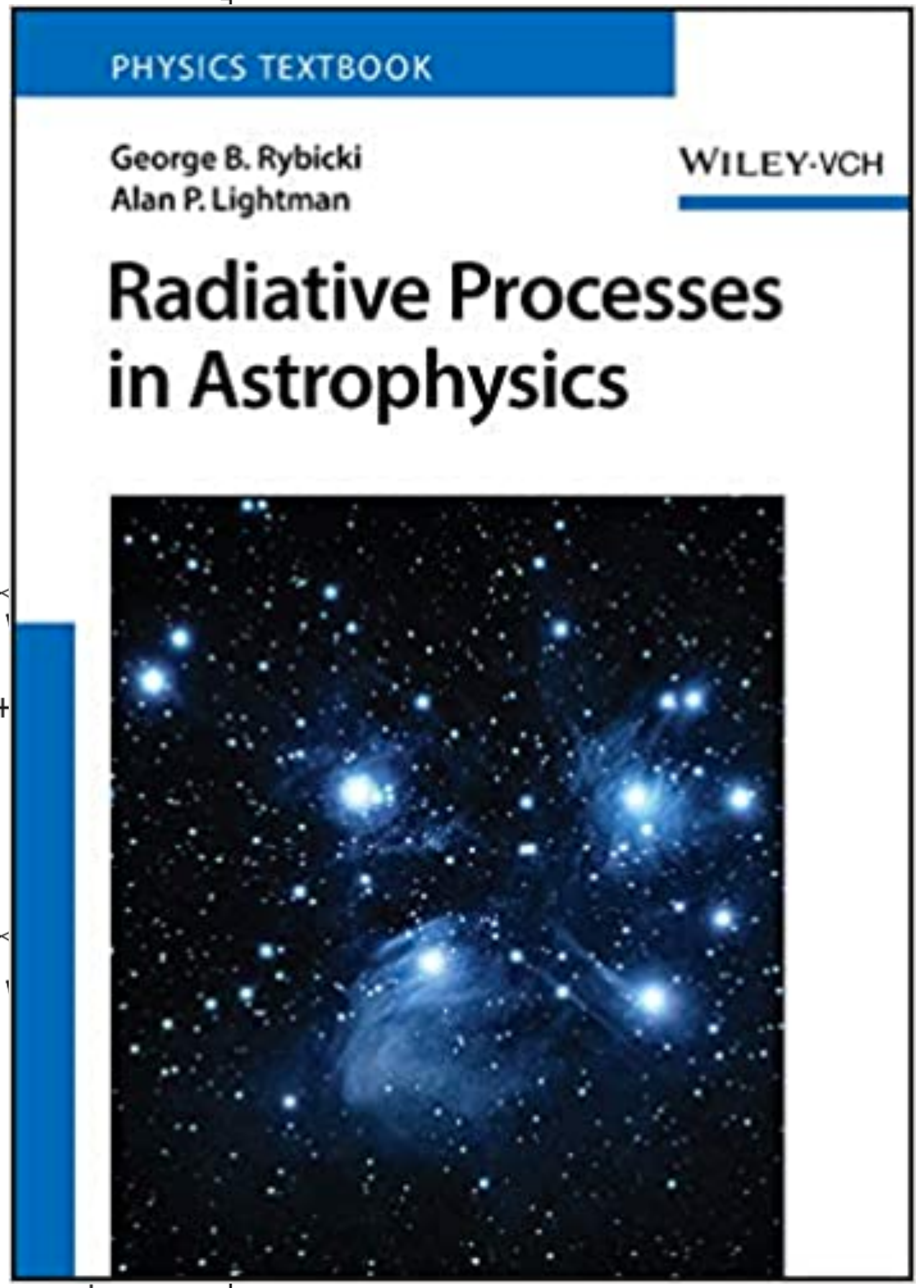
- Spectrum of radio emitting electrons with $\nu F_\nu \propto \nu^{0.7}$ is a common feature of most PWNe
- An isotropic distribution of electrons would imply $dN/d\gamma \propto \gamma^{-p}$ with $p = 1.6$ (shocks give $p \geq 2$)
- What physical processes operating in the Nebula produce its emission spectrum?

Zanin 2017, Lyutikov et al 2019



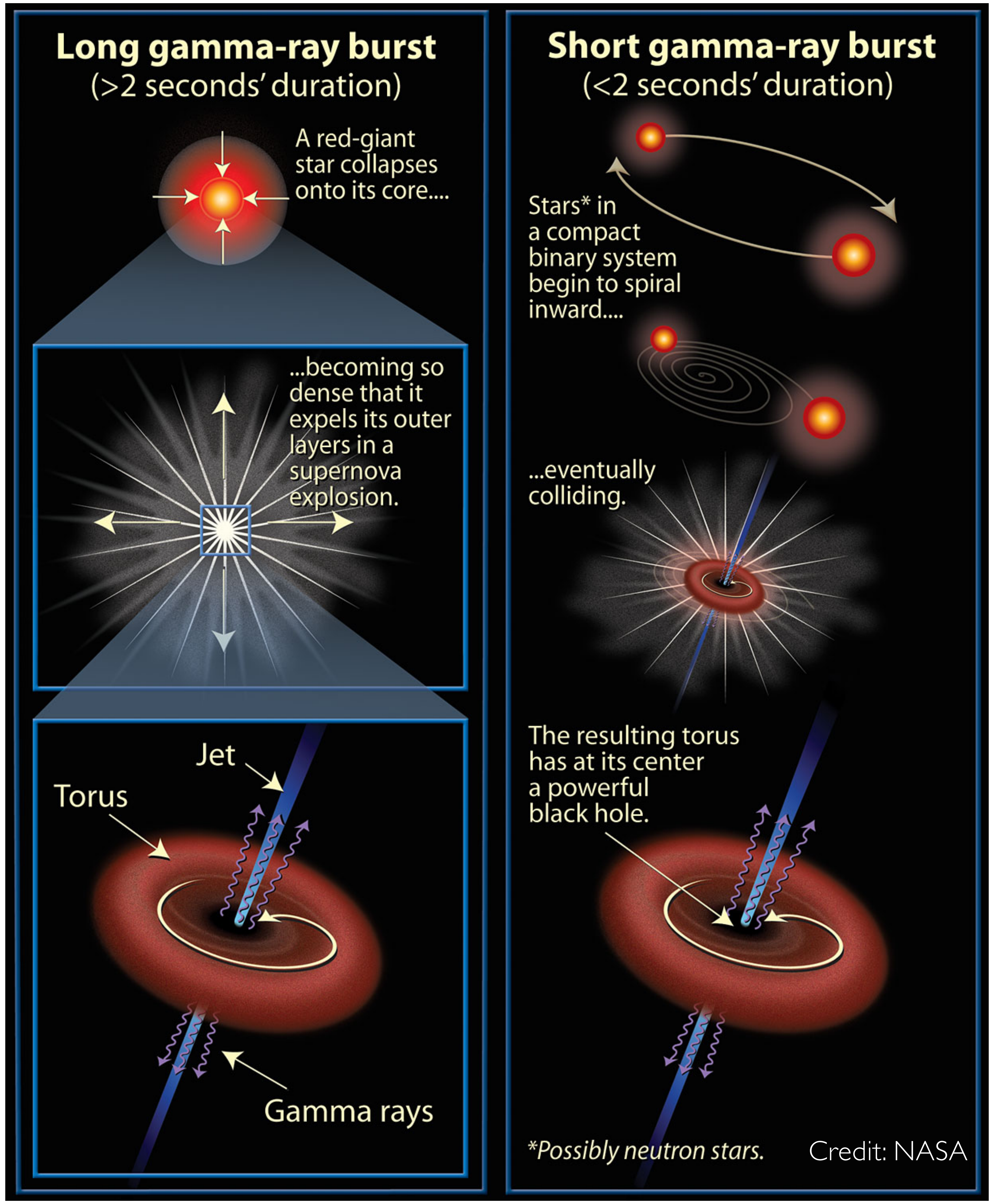
Properties of Gamma-Ray Bursts:



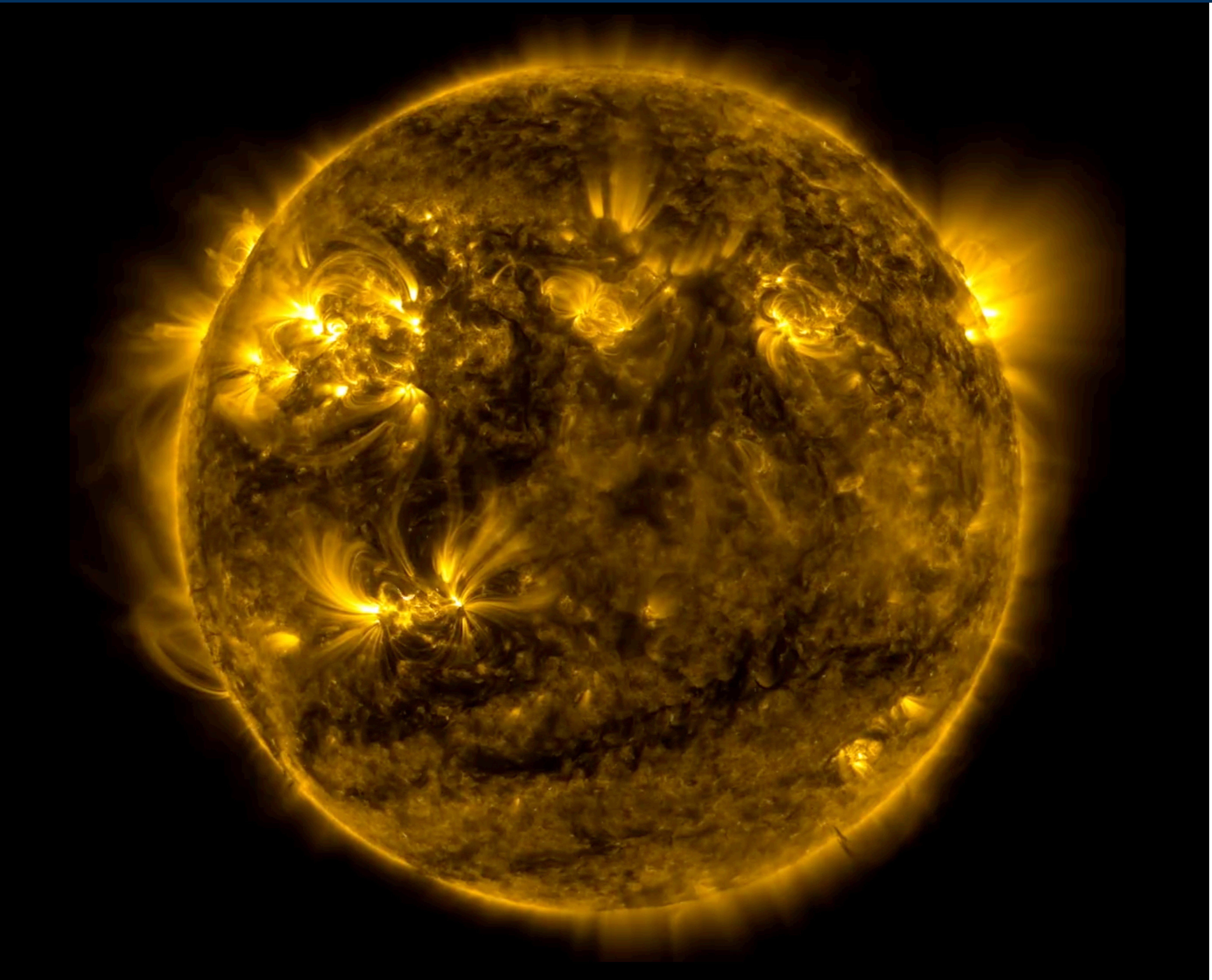


Steep spectrum: typically $s \sim 0.5 - 1.3$

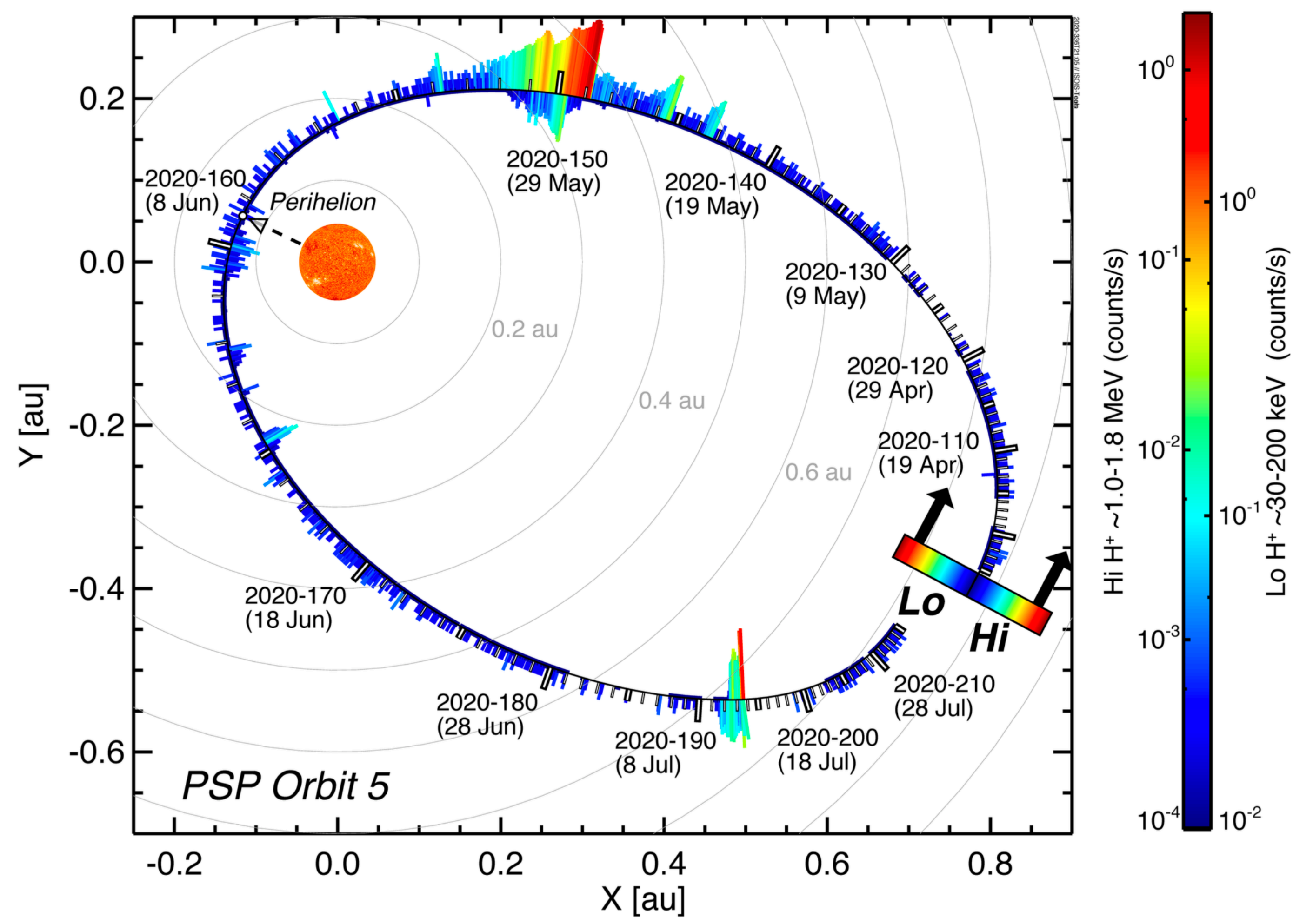
Theoretical limit for fast cooling electrons $s \leq 0.5$

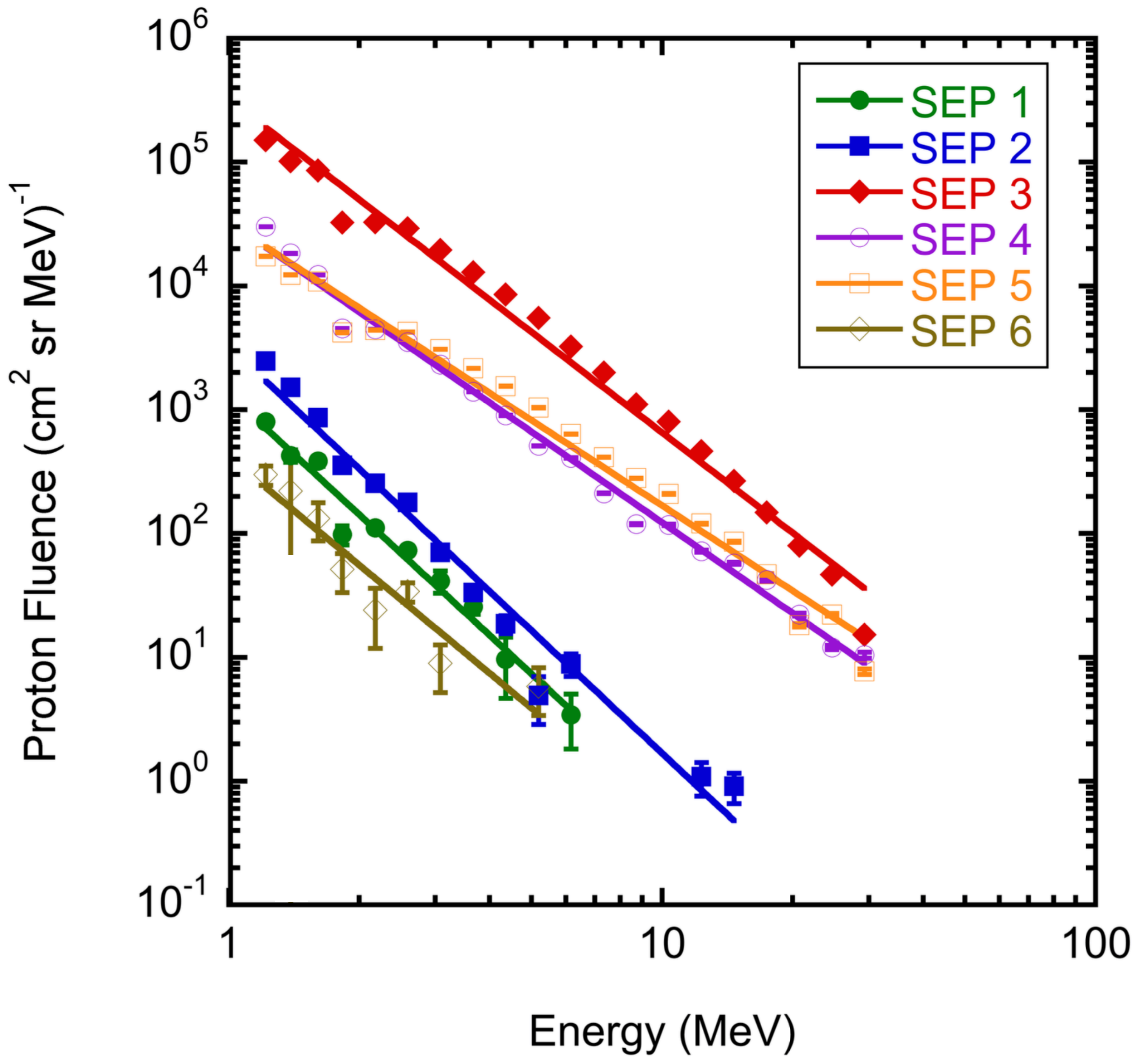


Nonrelativistic turbulence in the solar corona



Cohen et al. 2020





Working toward a solution of the problem: Timeline

Fermi Toy Model

Fermi 1949

Quasilinear Theory

Kennel & Engelmann 1966
Hall & Sturrock 1967
Lerche 1968
…….

Test-Particle Simulations

with prescribed fields

Brinca 1984
Decker & Vlahos 1986
…….

Test-Particle Simulations

with MHD fields

Ambrosiano+ 1988
Gray & Matthaeus 1992
Dmitruk+ 2003
…….

Hybrid Simulations

Servidio+ 2012
Kunz+ 2016
Arzamasskiy+ 2019
…….

Fully-Kinetic Simulations

relativistic regime

Zhdankin+ 2017
Comisso+ 2018
Nattila+ 2021
…….

Fully-Kinetic Simulations

nonrelativistic regime

Comisso & Sironi 2022

Fully kinetic treatment of the plasma

Evolution of the particle distribution function (for each species):

$$\frac{\partial f_s(\mathbf{x},\mathbf{p},t)}{\partial t} + \frac{d\mathbf{x}}{dt}\cdot\frac{\partial f_s(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{x}} + \frac{d\mathbf{p}}{dt}\cdot\frac{\partial f_s(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{p}} = 0$$

e.g., Lorentz force: $$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E}(\mathbf{x},t) + \frac{\mathbf{p}}{\gamma m_s c}\times\mathbf{B}(\mathbf{x},t)\right)$$

Can include other forces (e.g. radiation-reaction)

Evolution of the electromagnetic field:

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\cdot\mathbf{E} = 4\pi\rho = 4\pi\sum_s q_s\int f_s(\mathbf{x},\mathbf{p},t)d^3p$$

$$\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \qquad \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = \nabla\times\mathbf{B} - \frac{4\pi}{c}\mathbf{J} = \nabla\times\mathbf{B} - \frac{4\pi}{c}\sum_s q_s\int\frac{\mathbf{p}}{\gamma m_s}f_s(\mathbf{x},\mathbf{p},t)d^3p$$

Particle-in-cell approach on supercomputers

Solve Particle Equations of Motion:

$$\frac{d\boldsymbol{p}_p}{dt} = q_p\left(\boldsymbol{E} + \frac{\boldsymbol{p}_p}{\gamma_p m_p c} \times \boldsymbol{B}\right) + \boldsymbol{F}_{RR}$$

$$\frac{d\boldsymbol{x}_p}{dt} = \frac{\boldsymbol{p}_p}{\gamma_p m_p}$$

Extrapolate to Grid:

$$(\boldsymbol{x}, \boldsymbol{p})_p \to \boldsymbol{J}_i$$

Solve Maxwell's Equations

$$\frac{\partial \boldsymbol{E}}{\partial t} = c\,\boldsymbol{\nabla} \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\,\boldsymbol{\nabla} \times \boldsymbol{E}$$

Interpolate to Particles:

$$(\boldsymbol{E}, \boldsymbol{B})_i \to \boldsymbol{F}_p$$

PIC code: TRISTAN-MP
(Spitkovsky 2005)

Radiation-Reaction (reduced Landau-Lifshitz form):

$$\boldsymbol{F}_{RR} = \frac{2}{3} r_0^2 \Big[(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}) \times \boldsymbol{B} + (\boldsymbol{\beta} \cdot \boldsymbol{E})\boldsymbol{E}\Big] - \frac{2}{3} r_0^2 \gamma^2 \boldsymbol{\beta} \Big[(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B})^2 - (\boldsymbol{\beta} \cdot \boldsymbol{E})^2\Big]$$

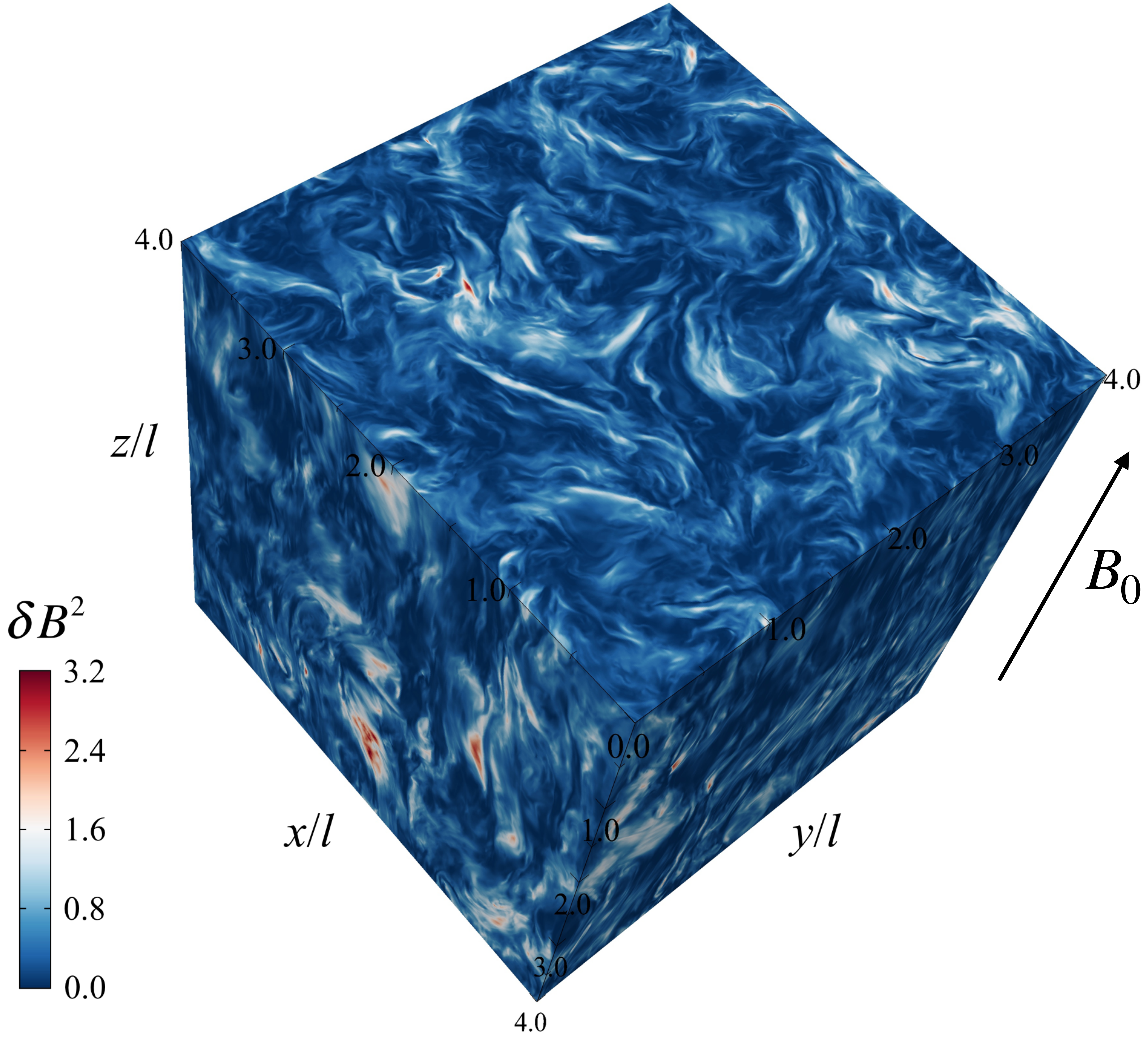
Theta (INCITE)


Pleiades (NASA)

Cori (NERSC)

+ Stampede2 (NSF), Ginsburg,…

Turbulence on supercomputers: simulations features [relativistic regime]



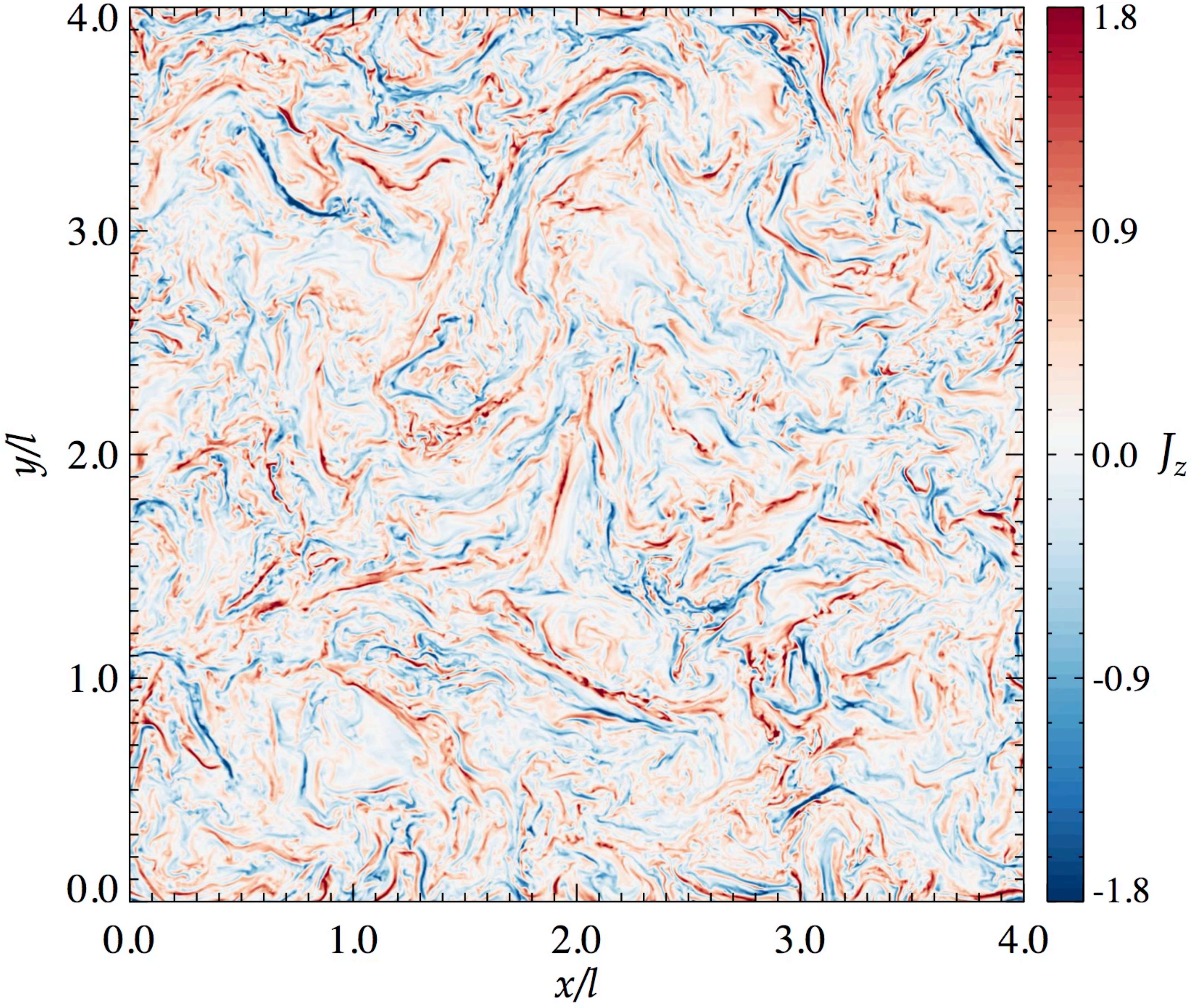
- 2460^3 cells; $\sim 2\times 10^{11}$ particles
- Turbulence seeded by initializing a spectrum of magnetic fluctuations
- Strong turbulence: $\delta B_0/B_0 = 1$
- electron-positron plasma: $m_i/m_e = 1$
- Physical size: $L^3 = (820 d_e)^3$
- High plasma magnetization:

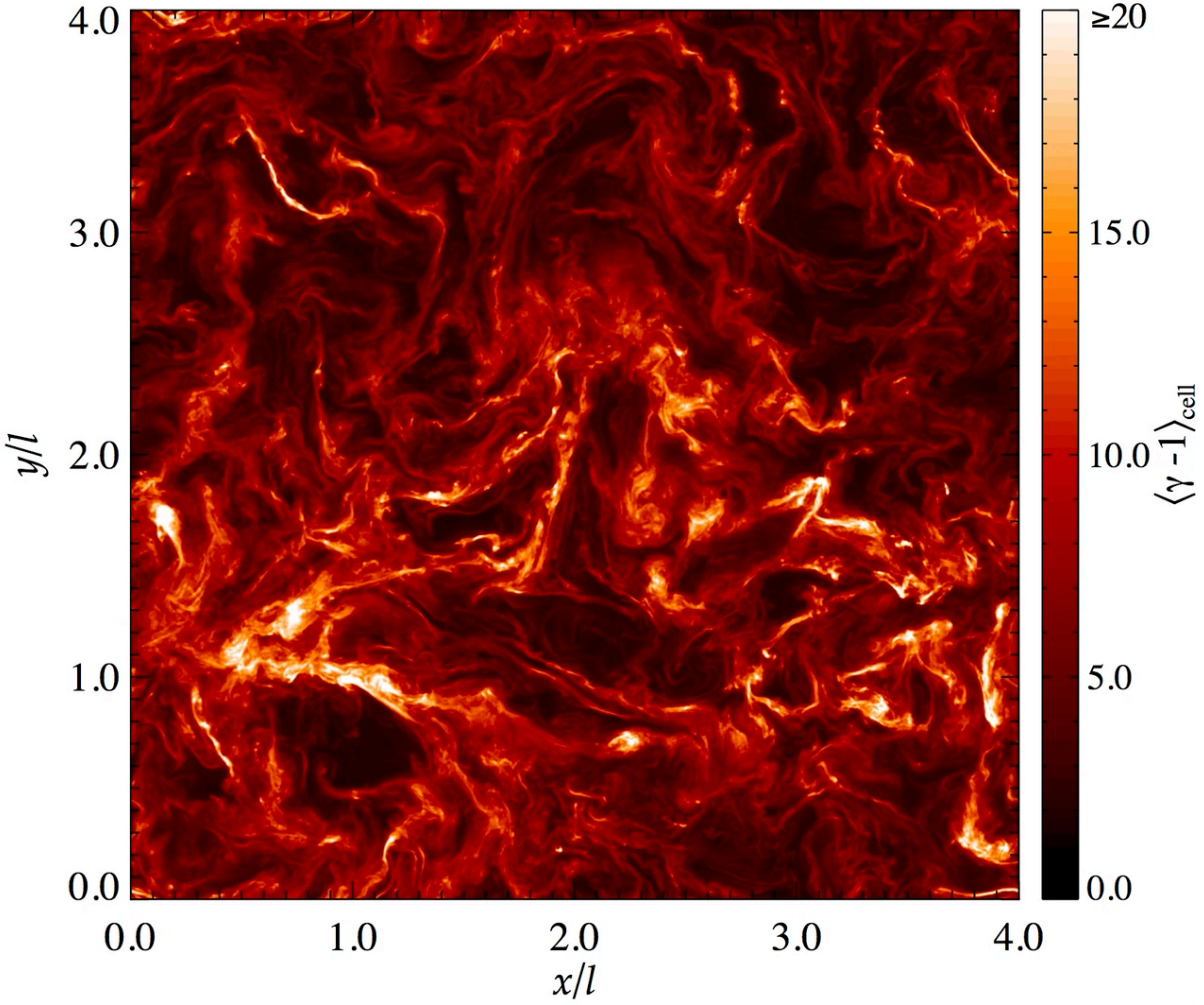
$$\sigma_0 = \frac{\delta B_0^2}{4\pi h_0} = 5 - 40$$

More details in Comisso and Sironi 2018/19

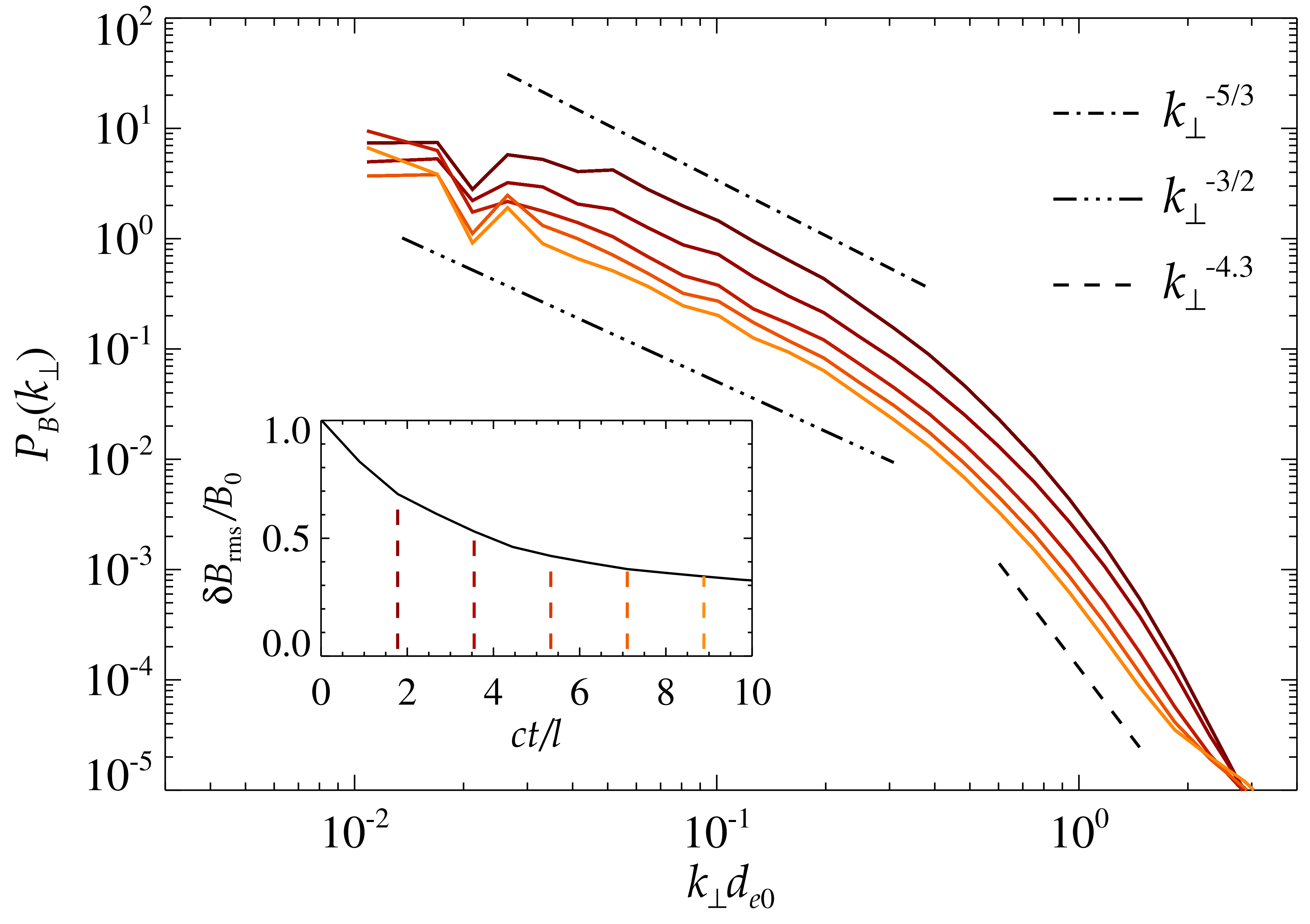
Relativistic turbulence [fly-through along the $B_0\hat{\mathbf{z}}$ direction]

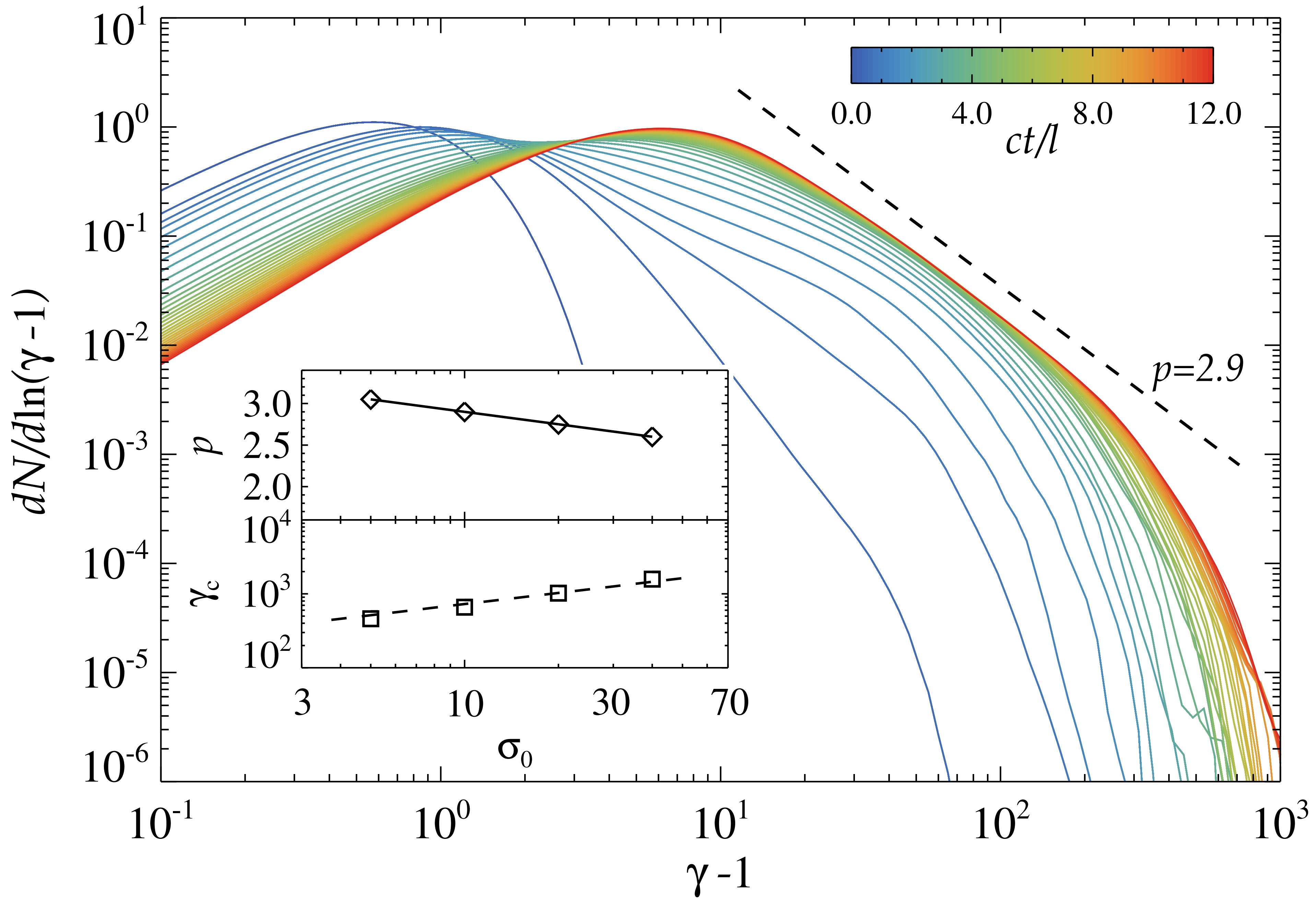
$$\delta B/B_0 = 1 \ , \ \sigma = 10 \ , \ L^3 = (820 d_e)^3, \ m_i/m_e = 1$$





Development of nonthermal power-law tails in relativistic turbulence





- Particle Heating with $\Delta\gamma \sim \frac{1}{2}\frac{\delta B_0^2}{8\pi n_0 m_e c^2} = \frac{\sigma_0}{4}$ $\quad\sigma_0 = \frac{\delta B_0^2}{4\pi\rho c^2}$

- Non-thermal Particle Acceleration with $\Delta\gamma \gg \sigma_0/4$ up to $\gamma_{cutoff} \sim (eB_0/m_e c^2)\, l$ and converged power law index $p = -d\log N/d\log\gamma$ Comisso+ 2018, '19, '20

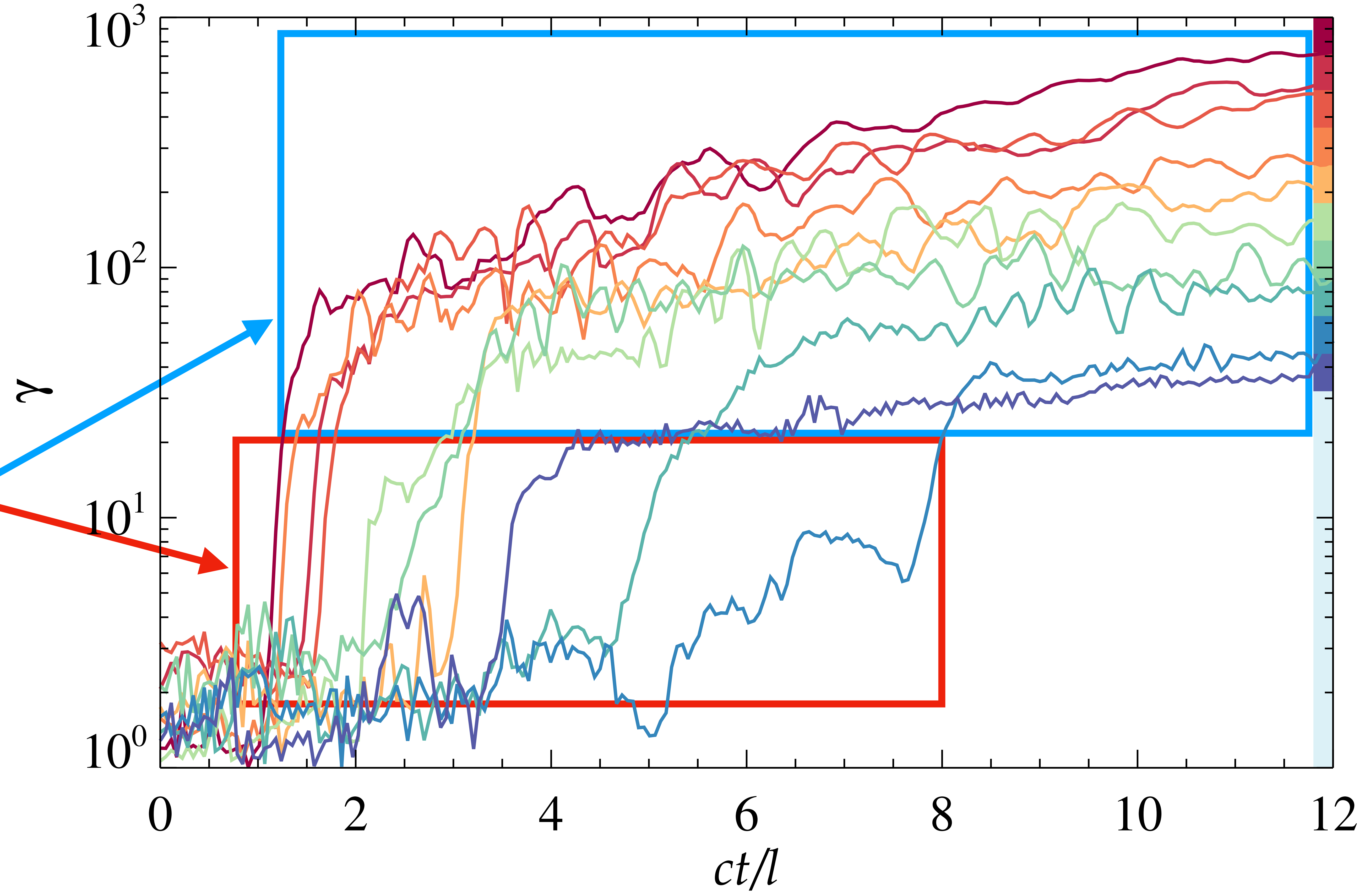
(see also Zhdankin+ 2018 for $\sigma_0 \sim 1$; Nättilä & Beloborodov 2021 for $\sigma_0 \gg 1$)

Particle energization as a two-stage process

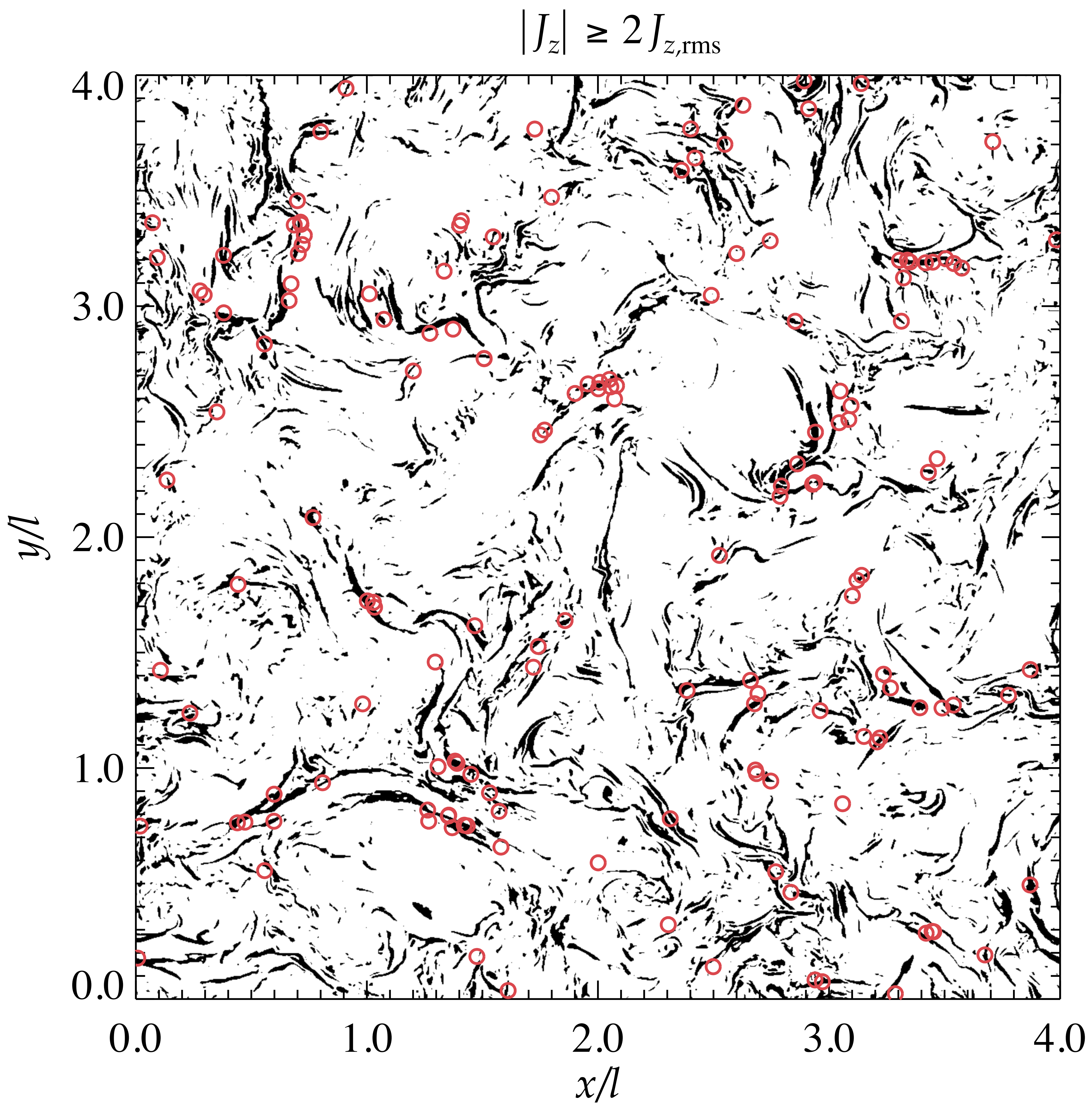
Particle energization occurs via a two-stage acceleration process:

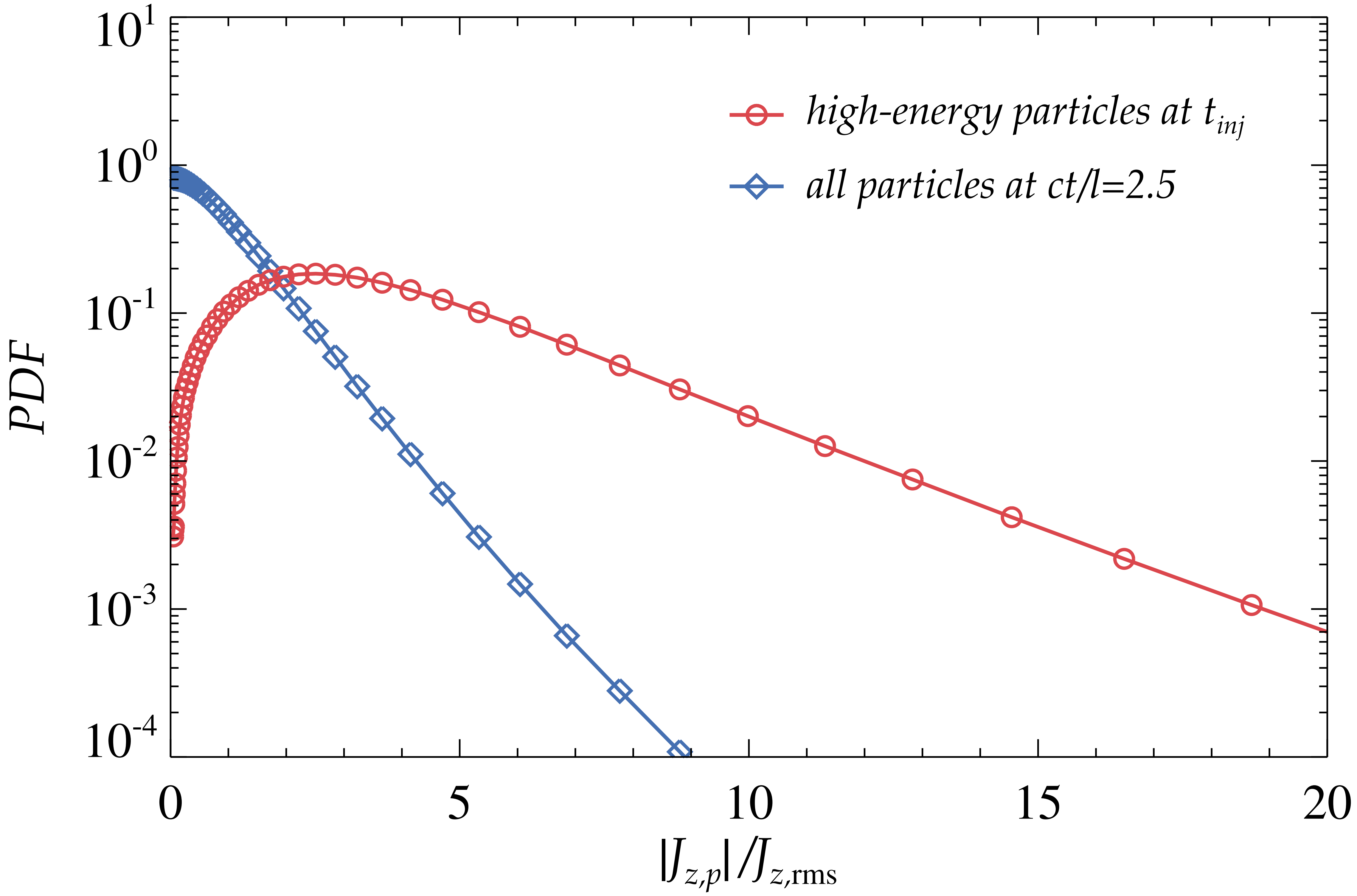
1. Particle Injection (via magnetic reconnection)

2. Stochastic Acceleration (via scatterings off larger scale turbulent fluctuations)



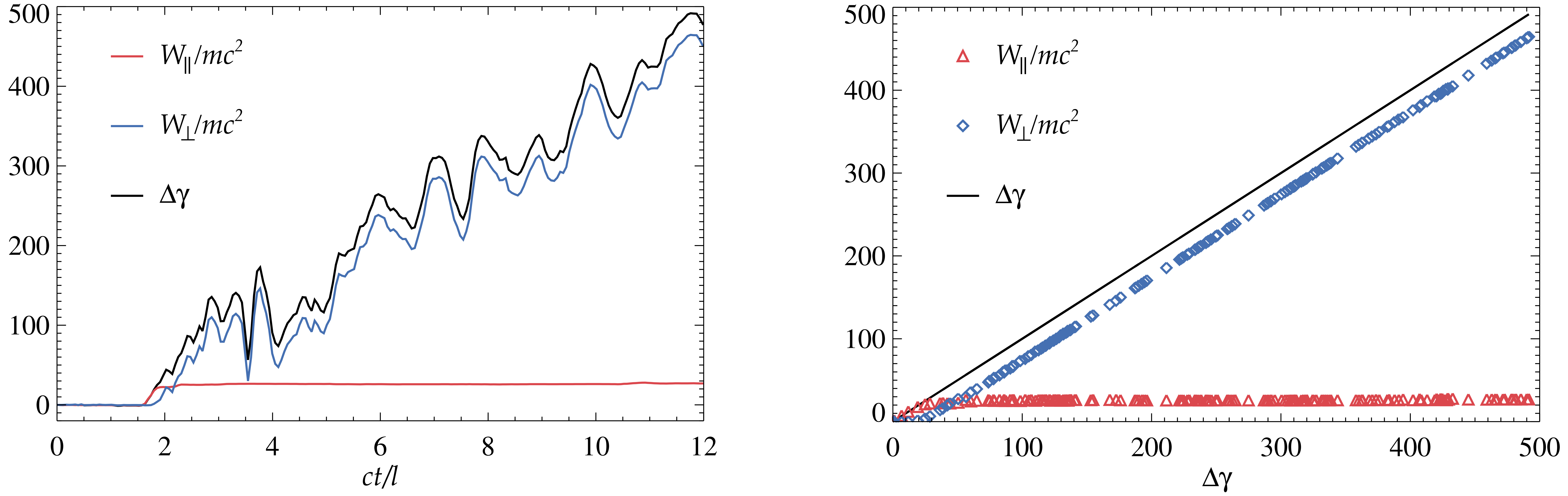
1st acceleration stage (particle “injection”)





- This acceleration occurs when particles experience high $|J_z|$

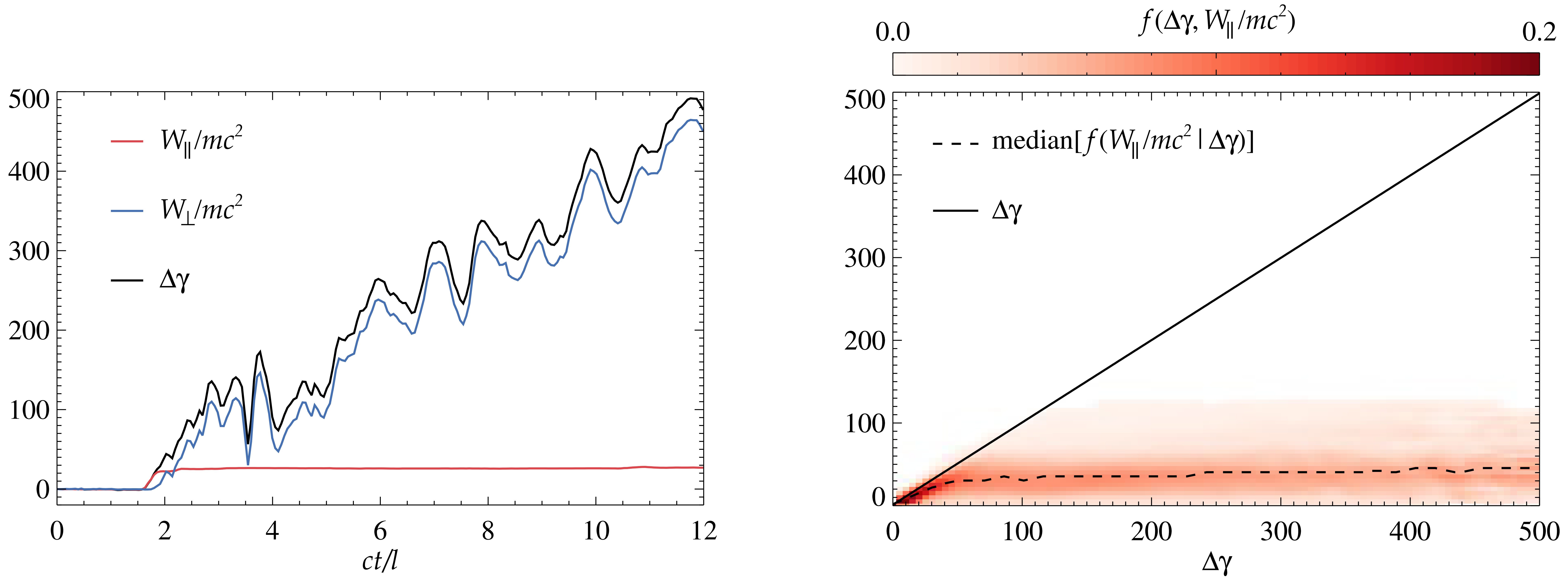
1st acceleration stage ("injection" via magnetic reconnection)



- The injection stage gives $\Delta\gamma_{\rm inj} \sim 3\sigma_0$ (Comisso and Sironi 2018, 2019)
- $\mathbf{v} \cdot \mathbf{E}_{\parallel}$ energization is critical initially (low $\Delta\gamma$-range) $\longrightarrow$ $\frac{d\gamma}{dt} \sim \frac{e}{m_e c}\beta_{\rm rec}\delta B_{\rm rms}$

  $\mathbf{v} \cdot \mathbf{E}_{\perp}$ energization is responsible for further acceleration

1st acceleration stage (“injection” via magnetic reconnection)



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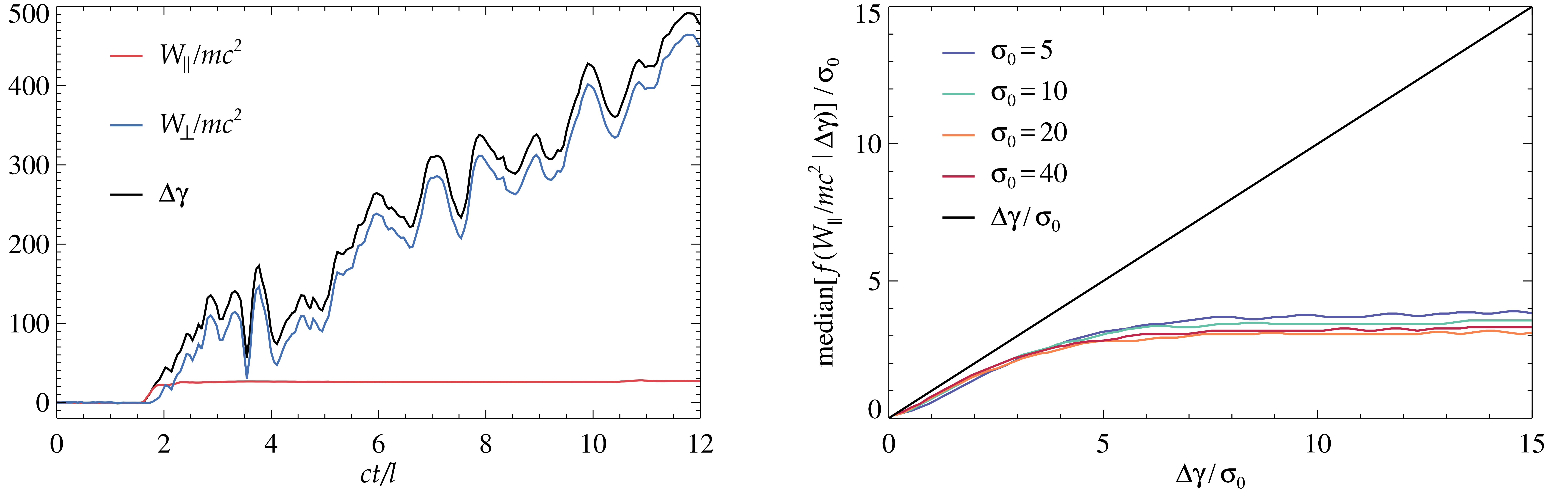

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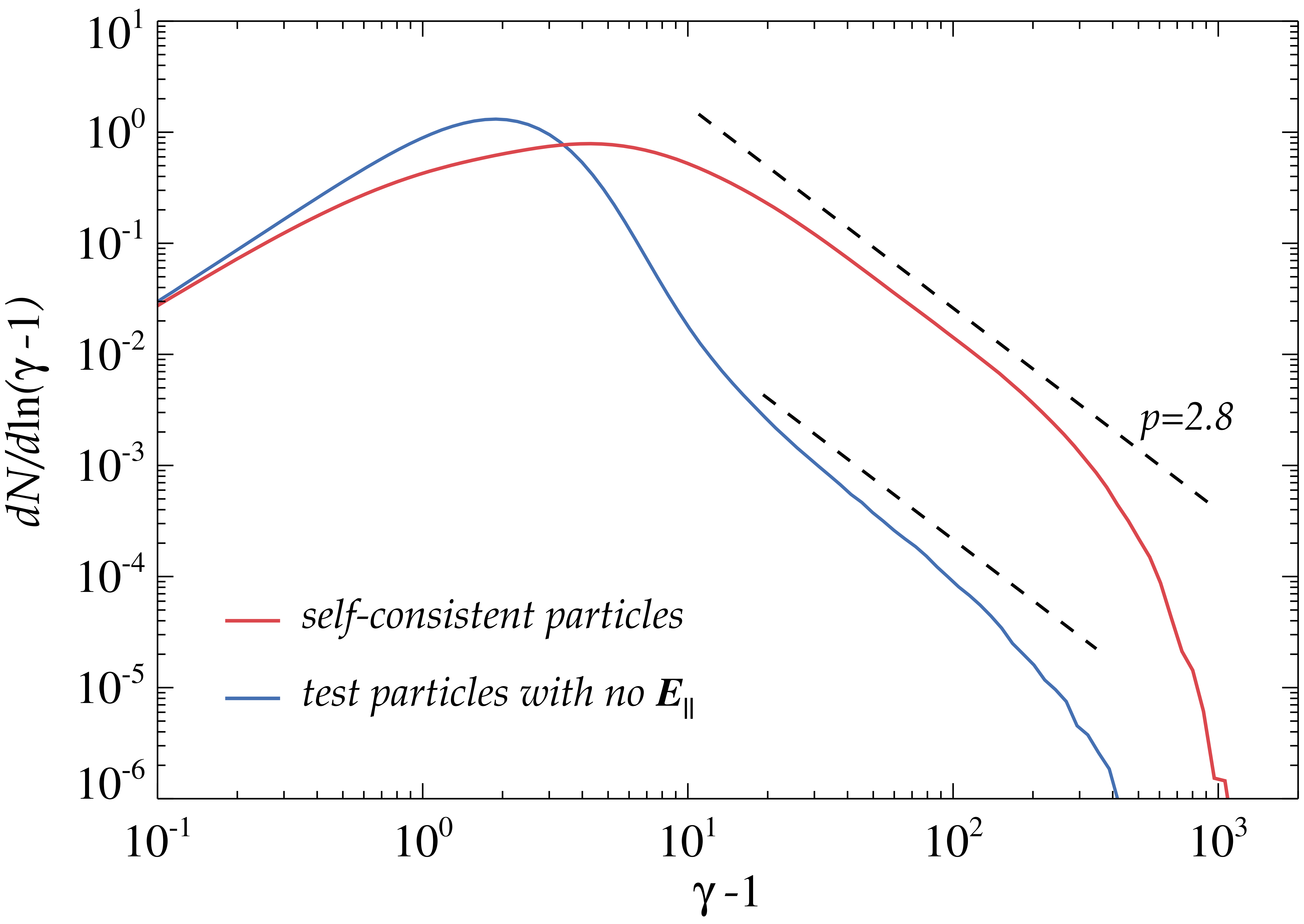
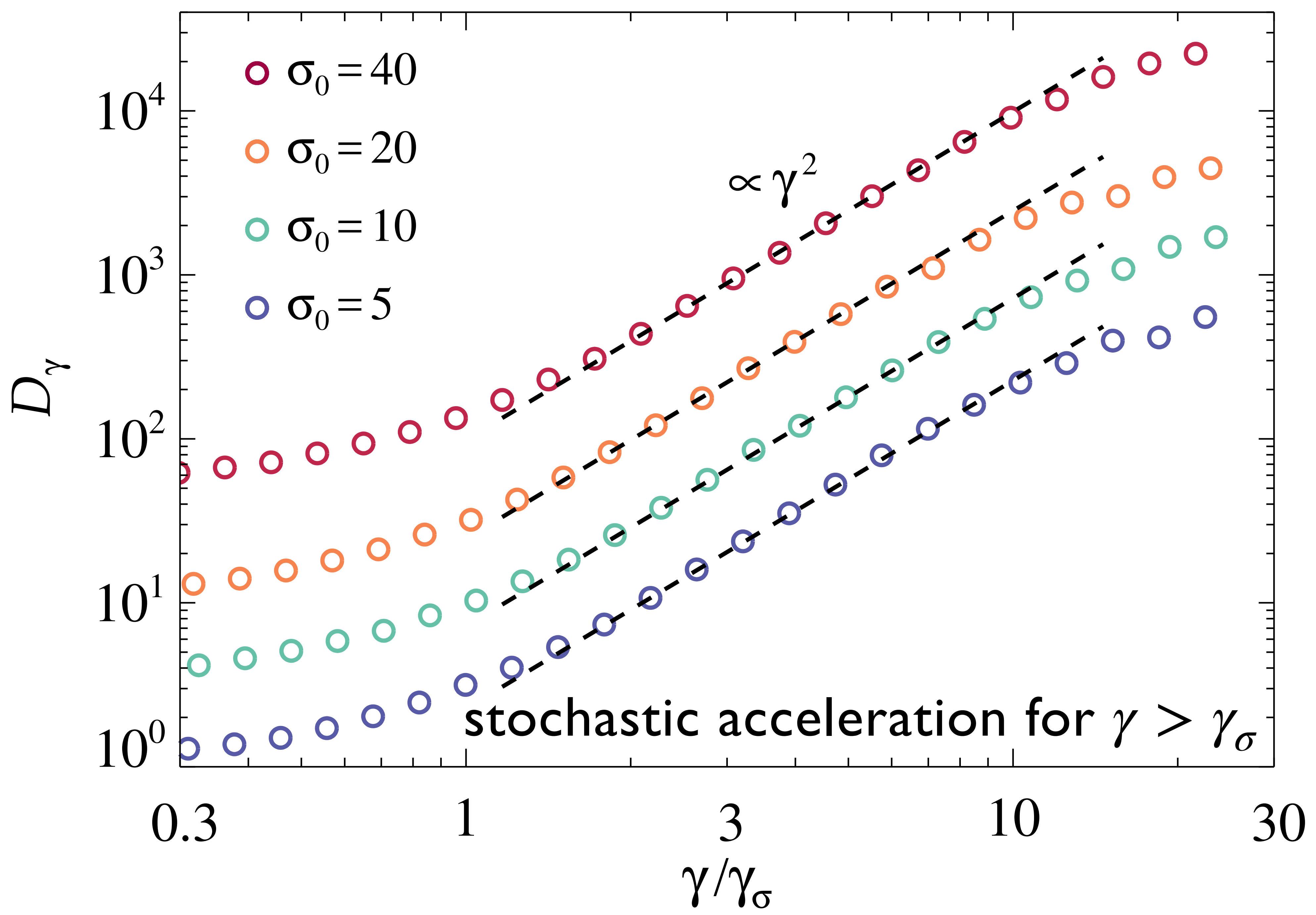
  $\mathbf{v} \cdot \mathbf{E}_{\perp}$ energization is responsible for further acceleration

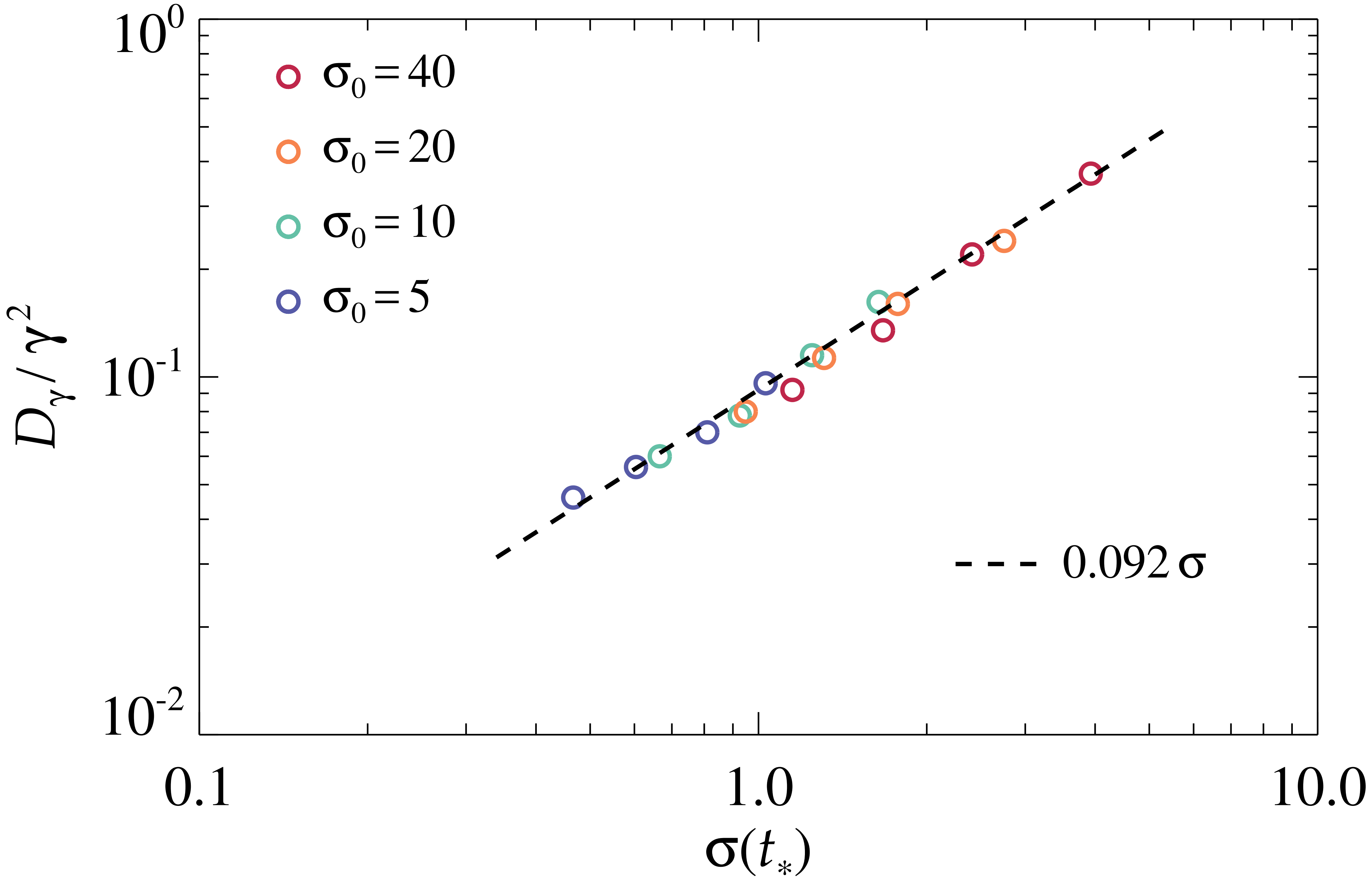
What if we artificially remove the magnetic-field-aligned electric fields?



- The normalization drops by 2 orders of magnitude (only $\sim 0.2\,\%$ of the particles in the nonthermal tail)

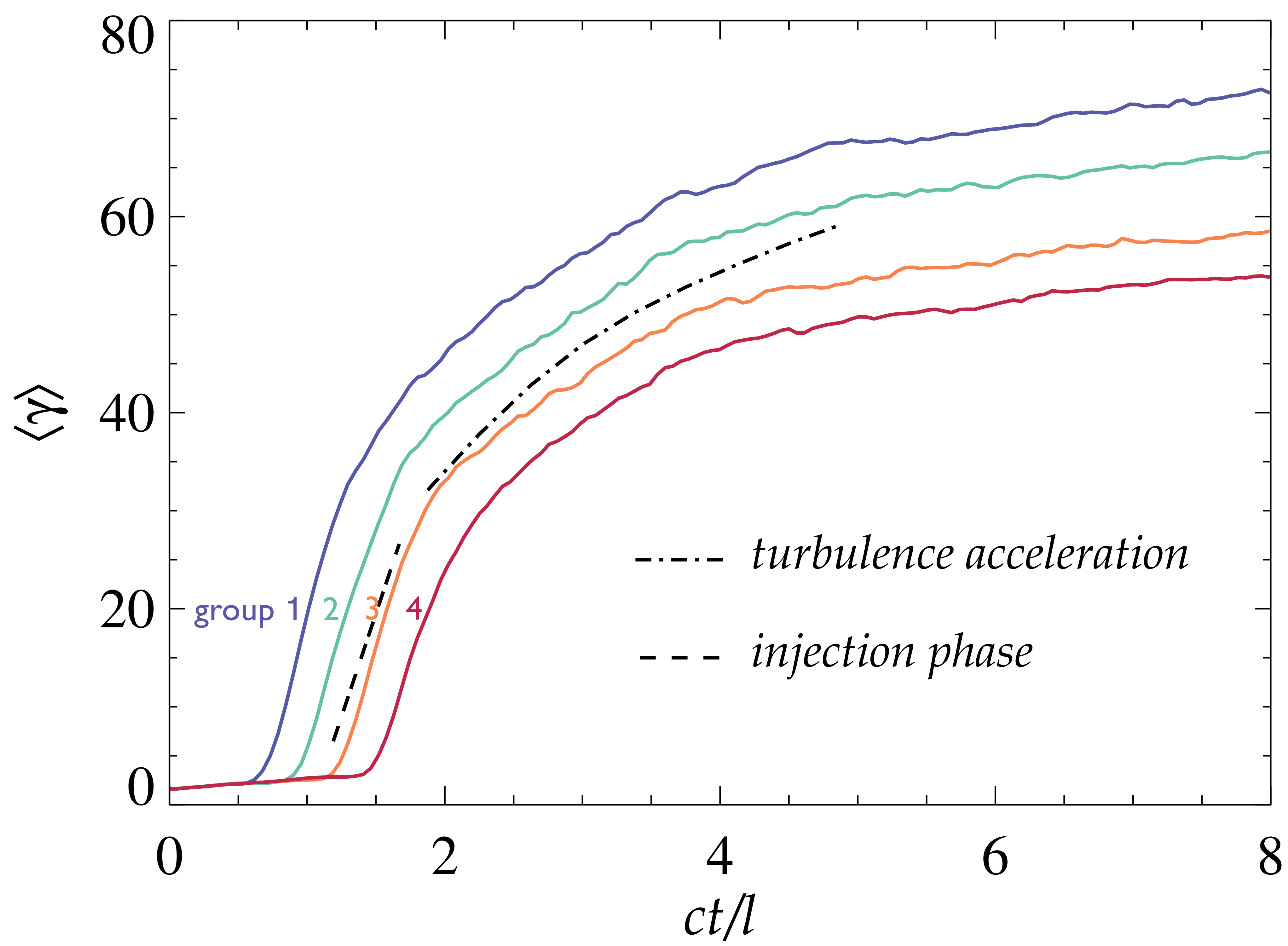
2nd acceleration stage (stochastic Fermi acceleration)

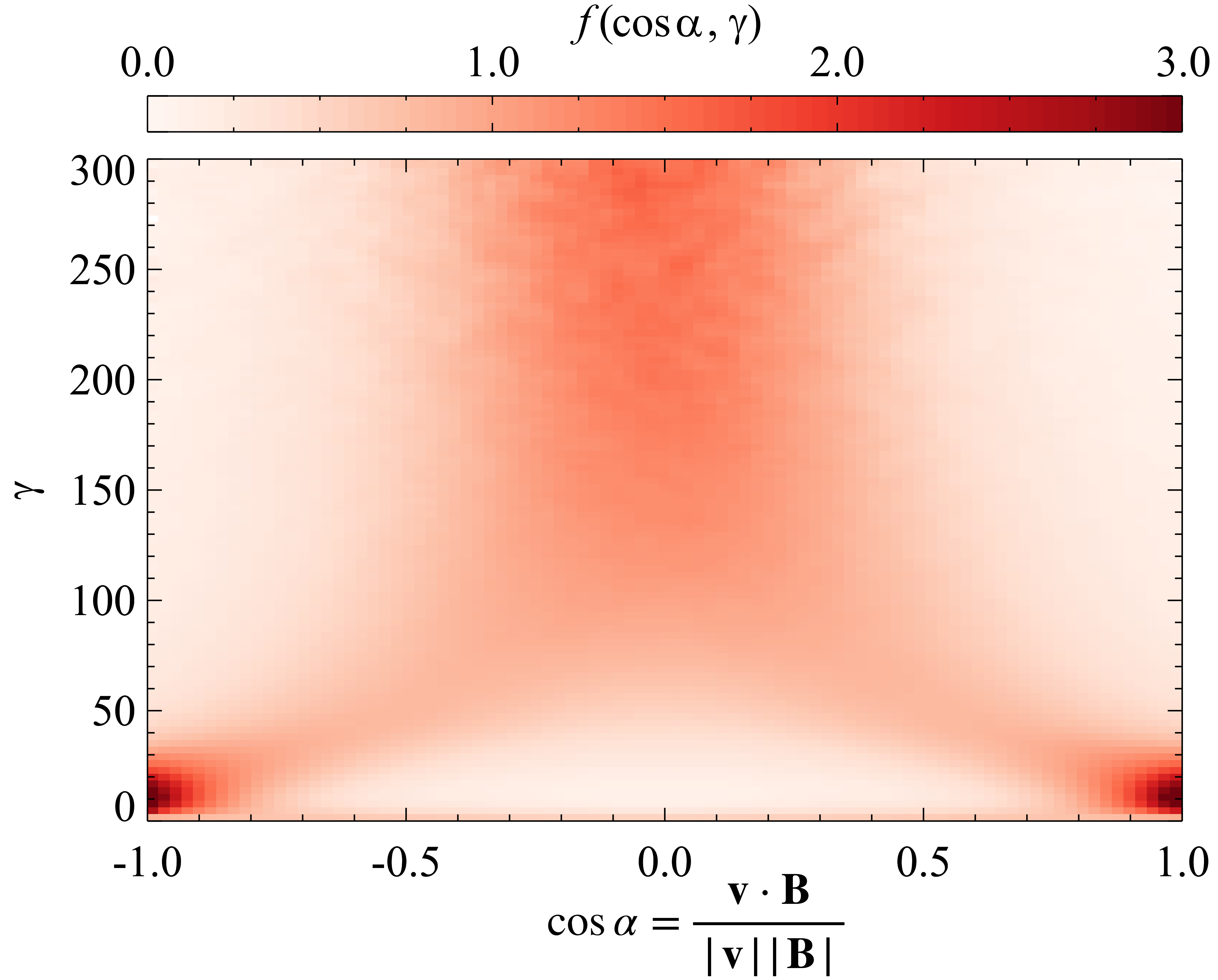




- Fokker-Planck equation in energy space: $$\frac{\partial f(\gamma,t)}{\partial t} = -\frac{\partial}{\partial \gamma}\left(A_\gamma f(\gamma,t)\right) + \frac{\partial^2}{\partial \gamma^2}\left(D_\gamma f(\gamma,t)\right)$$
- Mean rate of change of γ due to stochastic accelerations: $$A_\gamma = \frac{d\langle\gamma\rangle}{dt} = \frac{1}{\gamma^2}\frac{\partial}{\partial\gamma}\left(\gamma^2 D_\gamma\right)$$
- PIC simulations give $$D_\gamma \sim 0.1\sigma\left(\frac{c}{l}\right)\gamma^2 \longrightarrow \frac{d\langle\gamma\rangle}{dt} \sim 0.4\sigma\left(\frac{c}{l}\right)\gamma$$

Two-stage acceleration produces an *energy-dependent pitch-angle anisotropy*





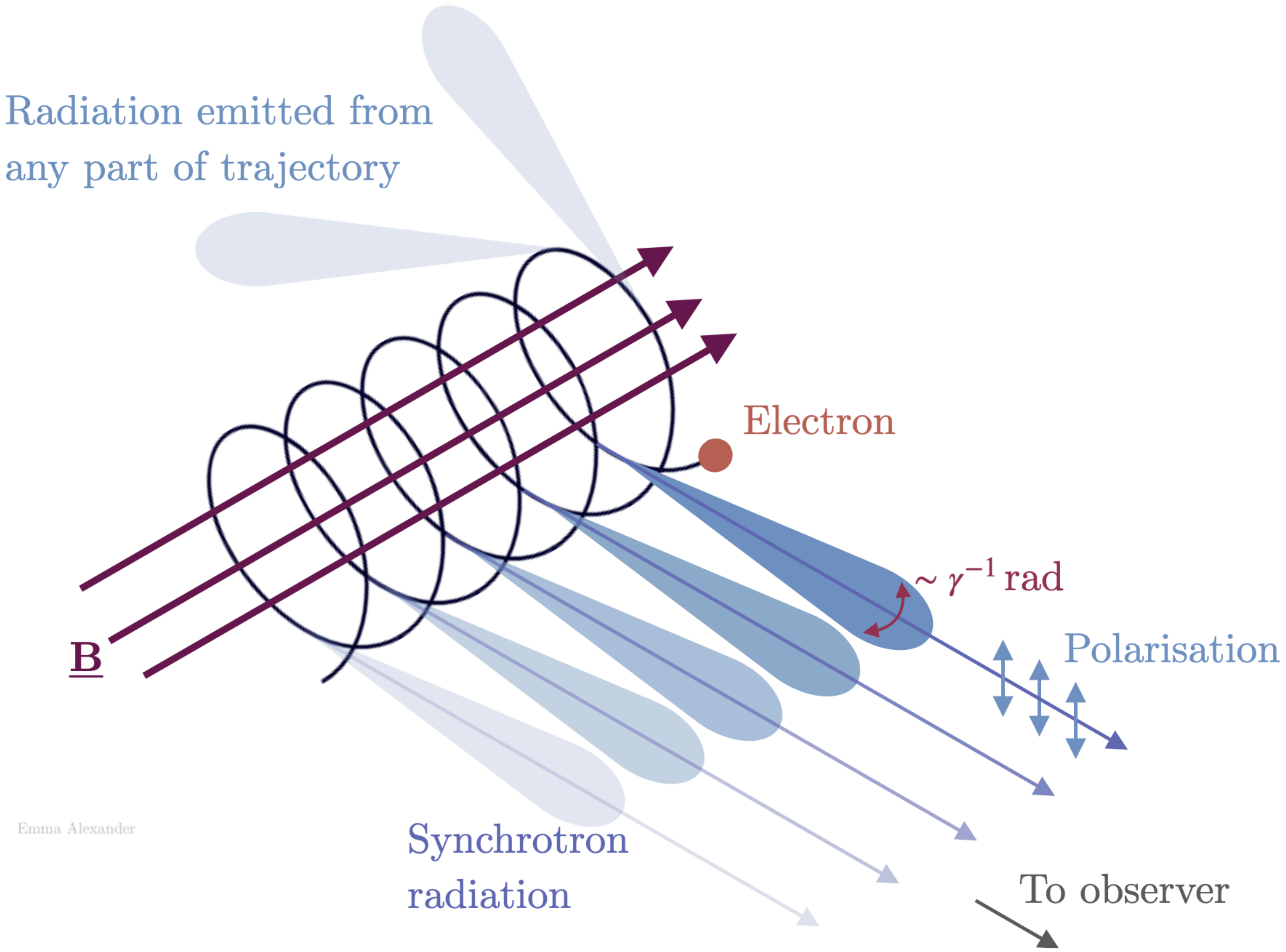
✓ injection phase: $\frac{d\gamma}{dt} \sim \frac{e}{m_e c}\beta_{rec}\delta B_{rms}$

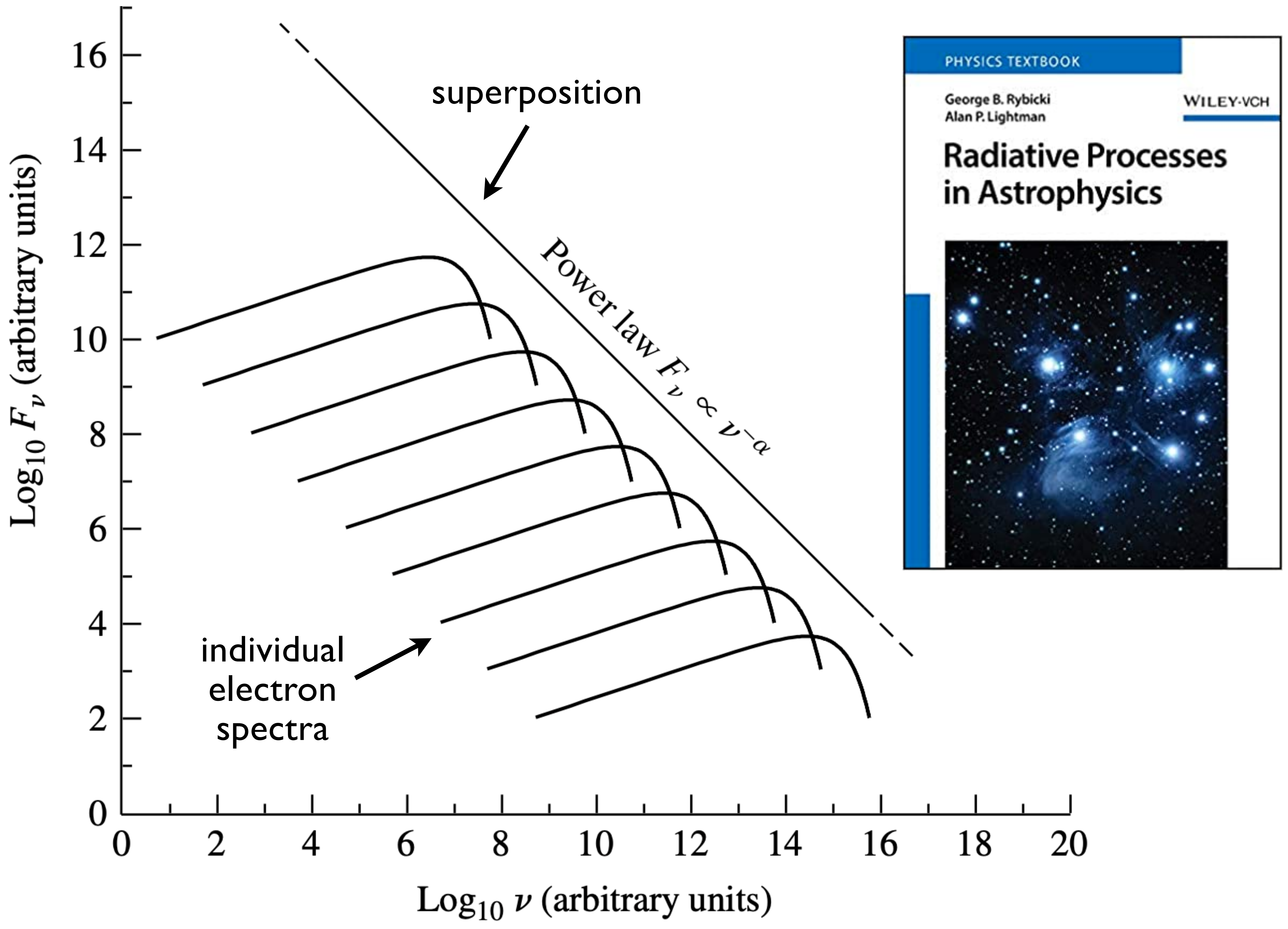
@ low-γ, $\mathbf{v}$ strongly anti/aligned with $\mathbf{B}$

✓ stochastic acceleration: $\frac{d\langle\gamma\rangle}{dt} \sim 0.4\sigma\left(\frac{c}{l}\right)\gamma$

@ high-γ, $\mathbf{v}$ mostly perpendicular to $\mathbf{B}$

Synchrotron radiation from the accelerated electrons





Synchrotron power radiated by one electron:

$$P_{\text{syn}} = 2\sigma_T c(B^2/8\pi)\gamma^2 \sin^2\alpha$$

Typical frequency of synchrotron photons:

$$\nu \sim \gamma^2 \nu_L \sin\alpha \quad (\nu_L = eB/2\pi m_e c)$$

Particles distributed as

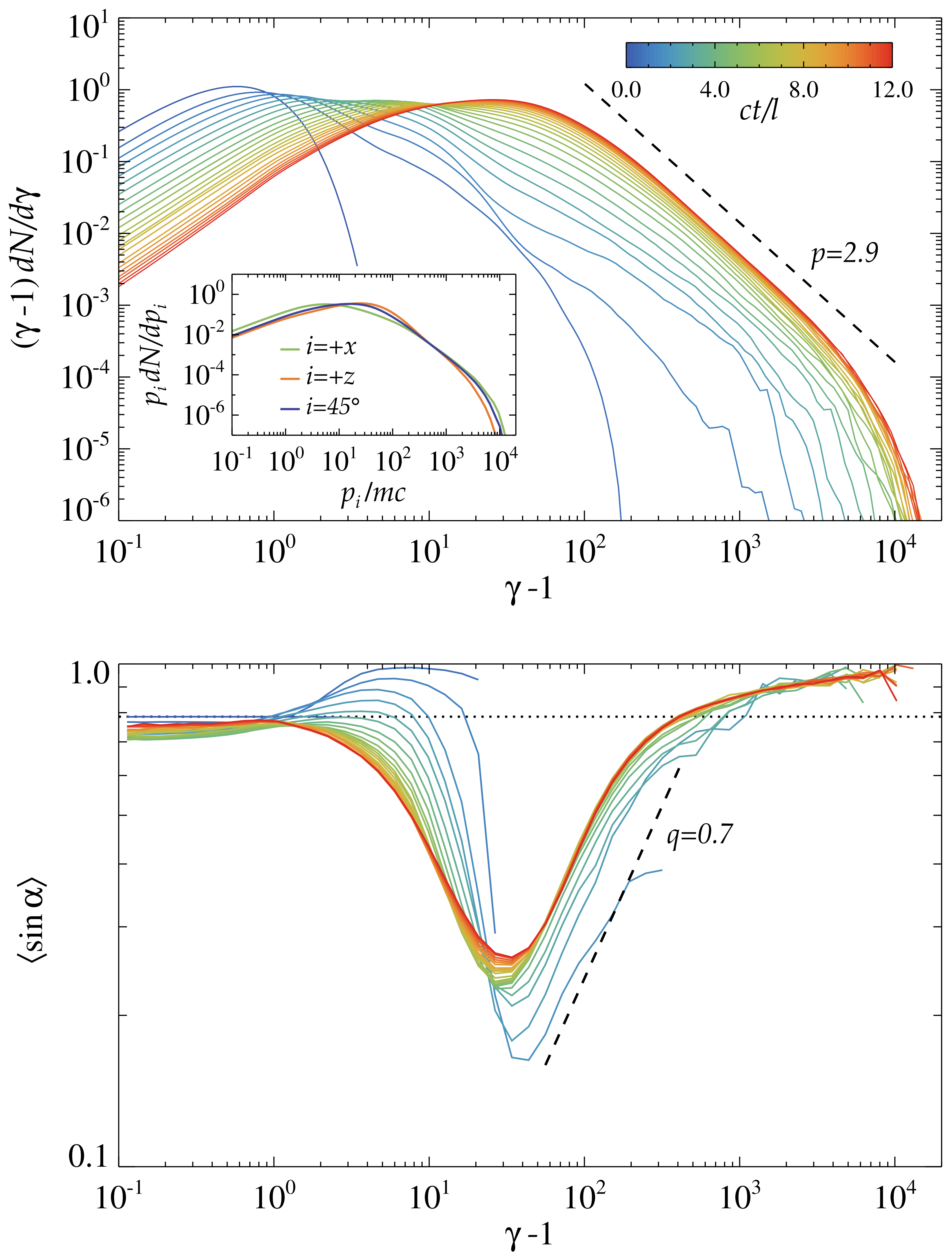
$$dN/d\gamma \propto \gamma^{-p}$$

lead to synchrotron energy flux

$$\nu F_\nu \propto \nu^{(3-p)/2}$$

(Hp: $\sin\alpha$ doesn't depend on γ)

Synchrotron spectrum accounting for energy-dependent pitch-angle anisotropy



For ultra-relativistic particles ($\gamma \gg 1$):

$$N_\gamma \sim \gamma(dN/d\gamma) \propto \gamma^{1-p}$$

$$P_{\rm syn} = 2\sigma_T c(B^2/8\pi)\gamma^2 \sin^2\alpha \propto \gamma^{2+2q}$$

$$\nu F_\nu \sim N_\gamma P_{\rm syn} \propto \gamma^{3-p+2q}$$

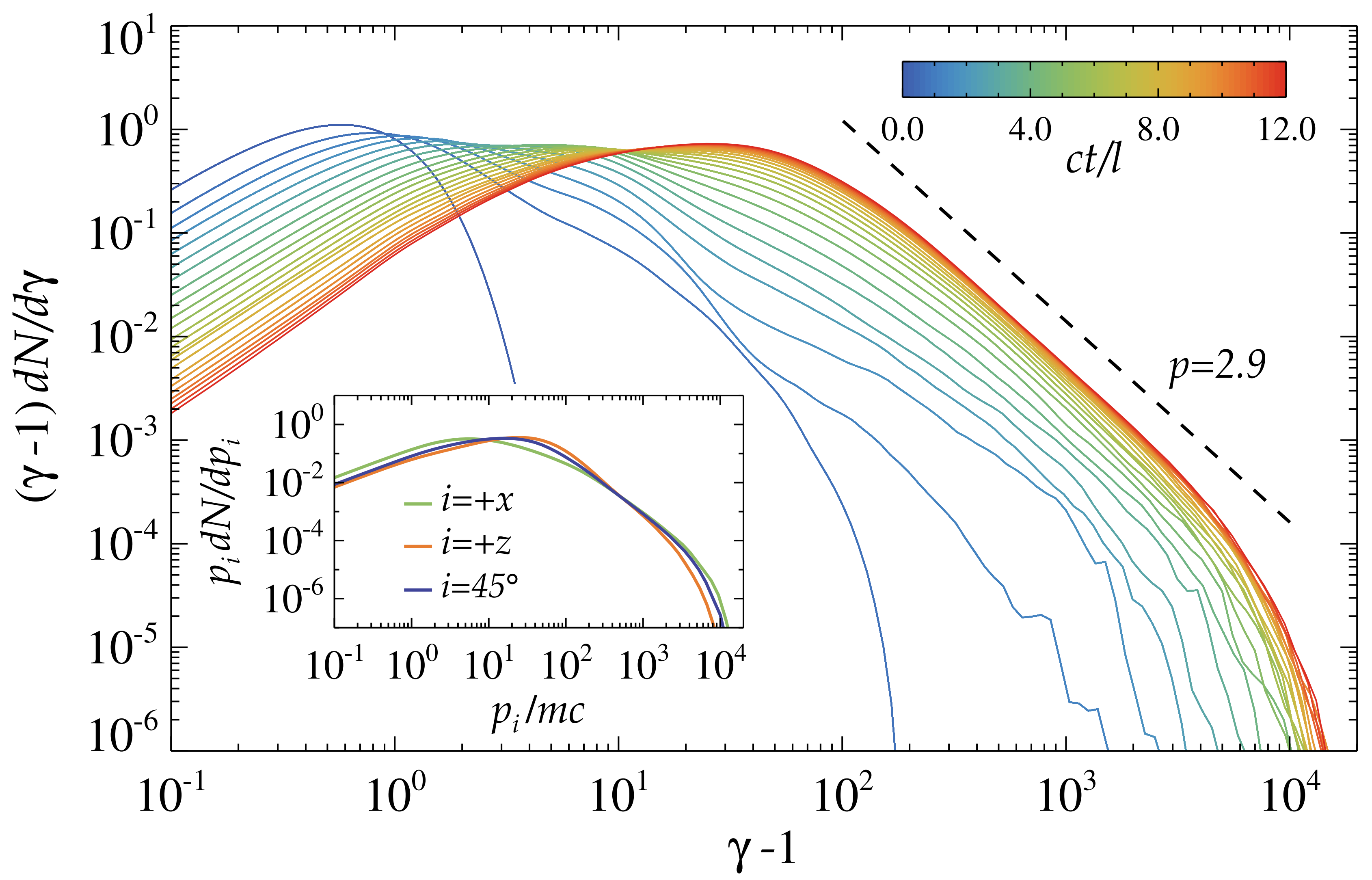
$$\nu \sim \gamma^2 \nu_L \sin\alpha \qquad (\nu_L = eB/2\pi m_e c)$$

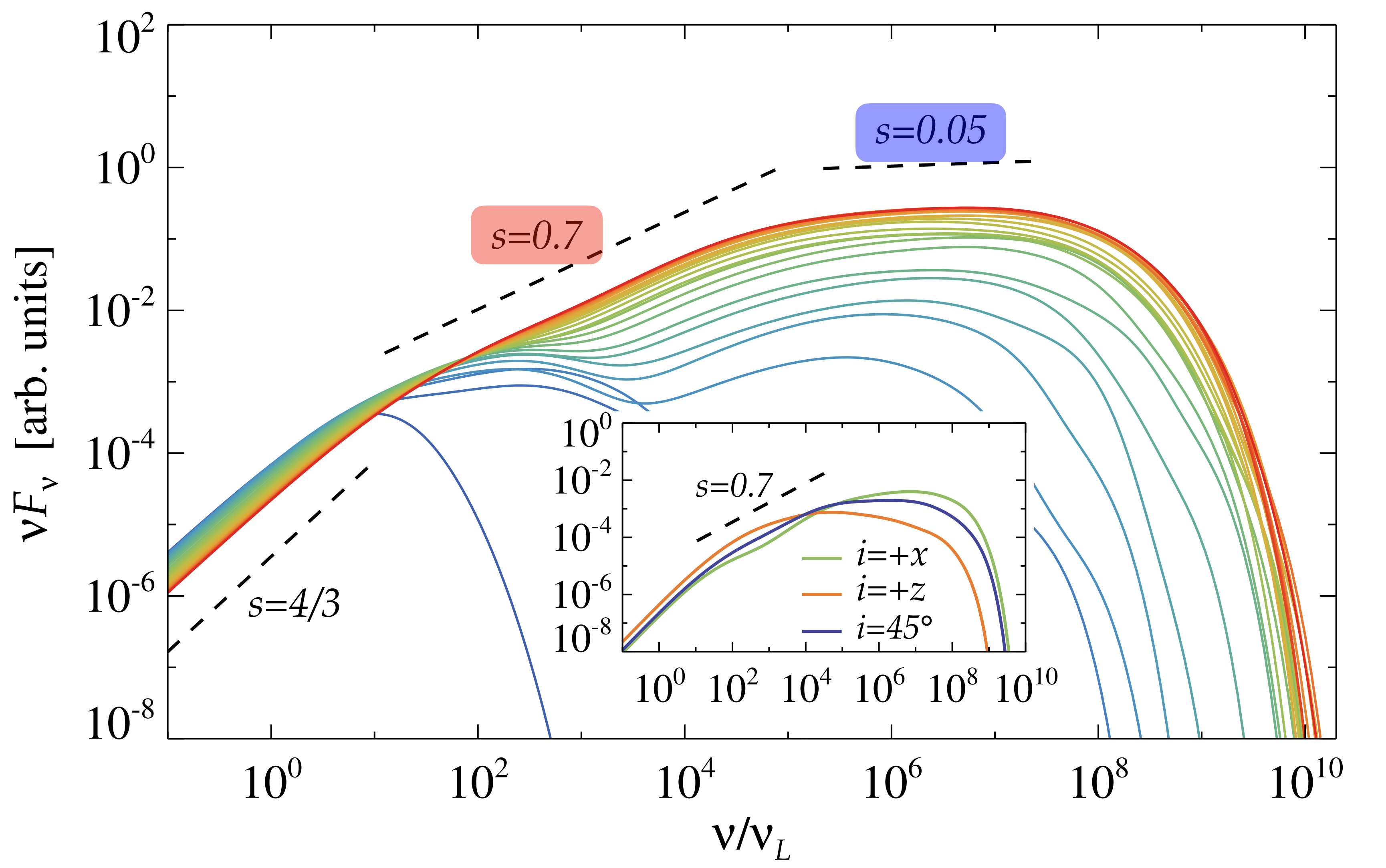
$$\nu F_\nu \propto \nu^{(3-p+2q)/(2+q)} \qquad \text{for } \nu_{\rm min} < \nu < \nu_{\rm crit} \sim \gamma_{\rm crit}^2 \nu_L$$

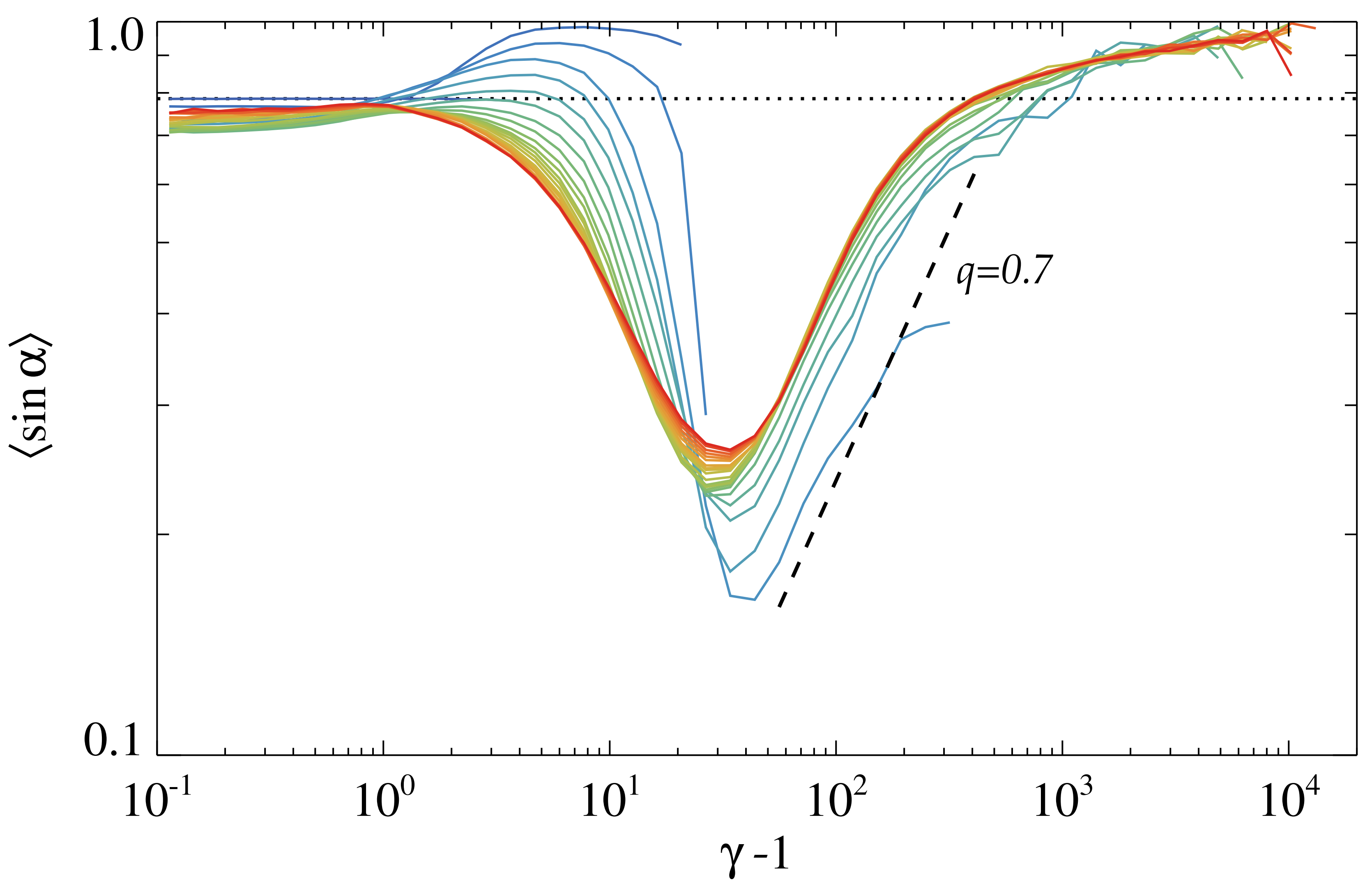
$$\nu F_\nu \propto \nu^{(3-p)/2} \qquad \text{for } \nu_{\rm crit} < \nu < \nu_{\rm max} \sim \gamma_{\rm max}^2 \nu_L$$

Comisso et al. 2020

Synchrotron spectrum accounting for energy-dependent pitch-angle anisotropy







$\nu \sim \gamma^2 \nu_L \sin\alpha$ $(\nu_L = eB/2\pi m_e c)$

$\nu F_\nu \propto \nu^{(3-p+2q)/(2+q)}$ for $\nu_{\rm min} < \nu < \nu_{\rm crit} \sim \gamma^2_{\rm crit}\nu_L$

$\nu F_\nu \propto \nu^{(3-p)/2}$ for $\nu_{\rm crit} < \nu < \nu_{\rm max} \sim \gamma^2_{\rm max}\nu_L$

Comisso et al. 2020

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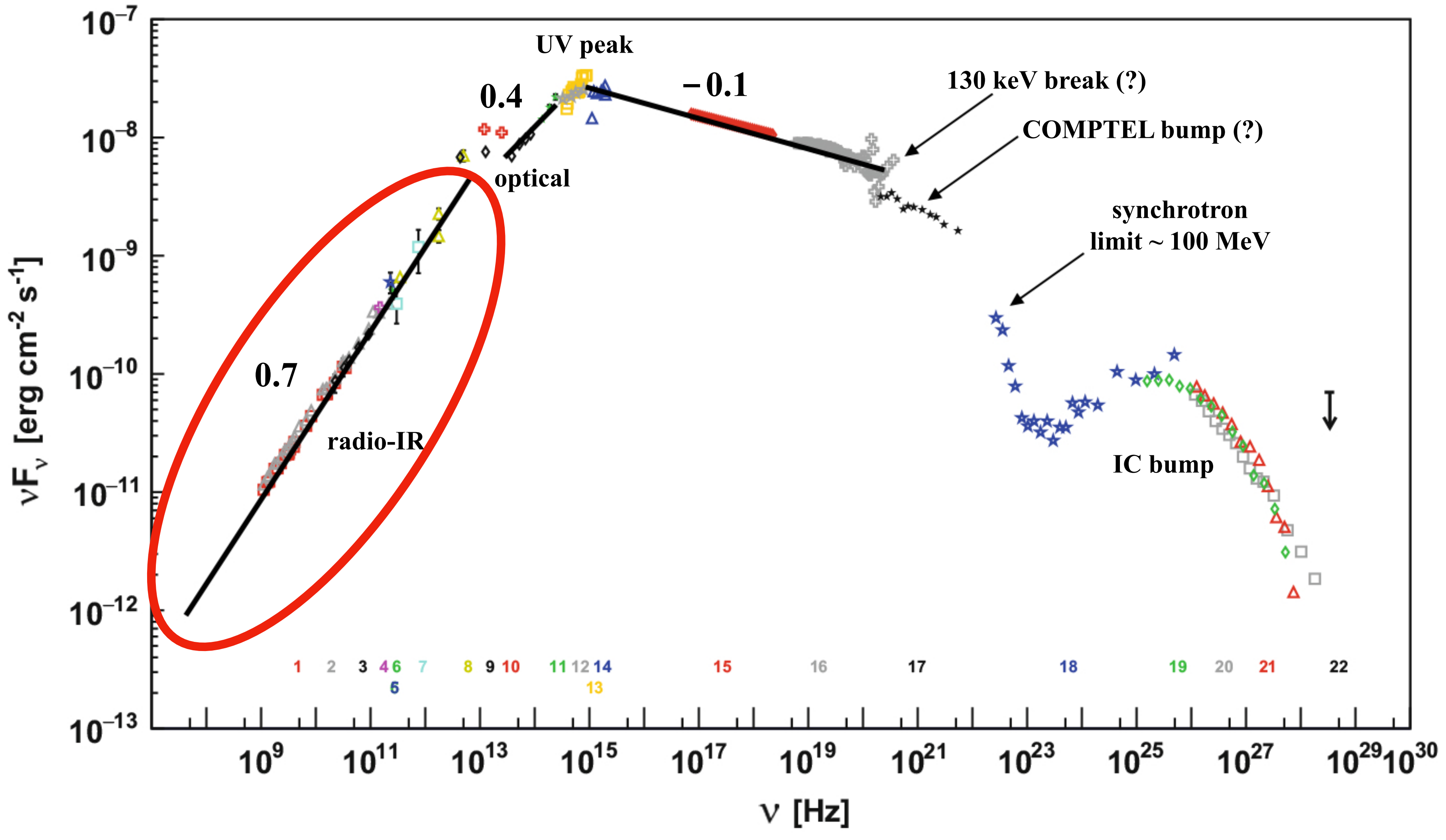

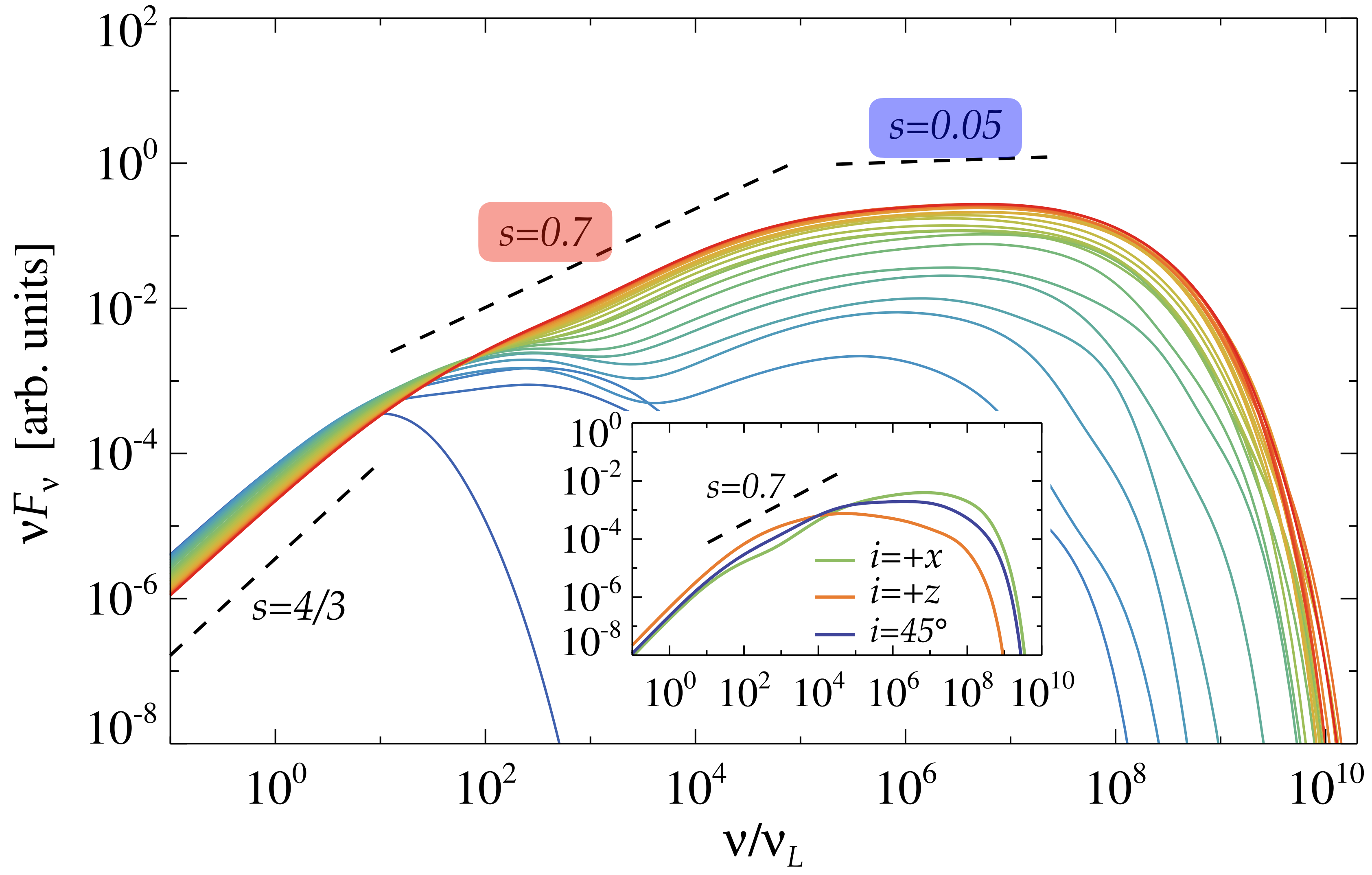

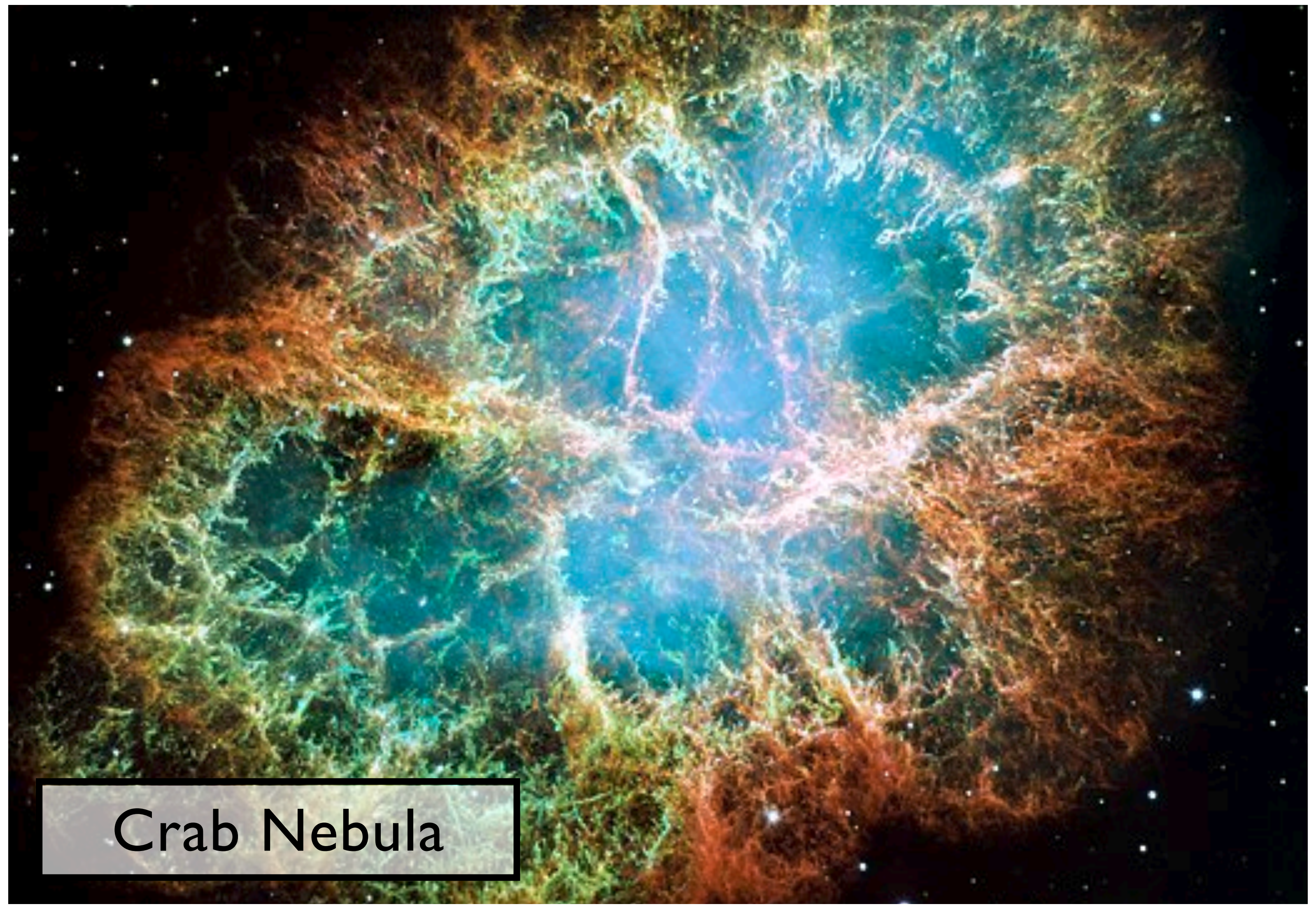

$\nu \sim \gamma^2 \nu_L \sin\alpha \qquad (\nu_L = eB/2\pi m_e c)$

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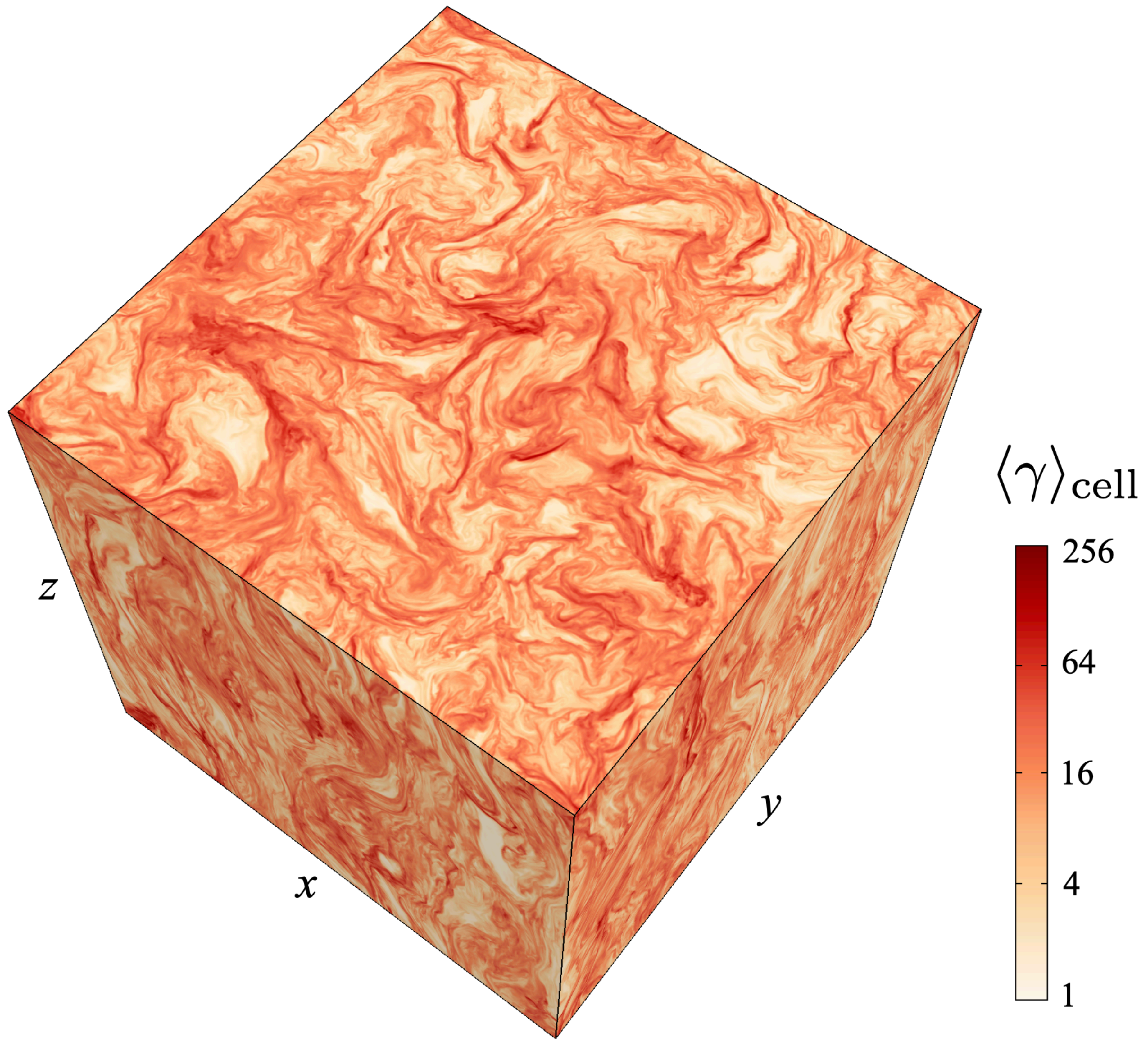
$\nu F_\nu \propto \nu^{(3-p)/2}$ for $\nu_{\rm crit} < \nu < \nu_{\rm max} \sim \gamma_{\rm max}^2 \nu_L$

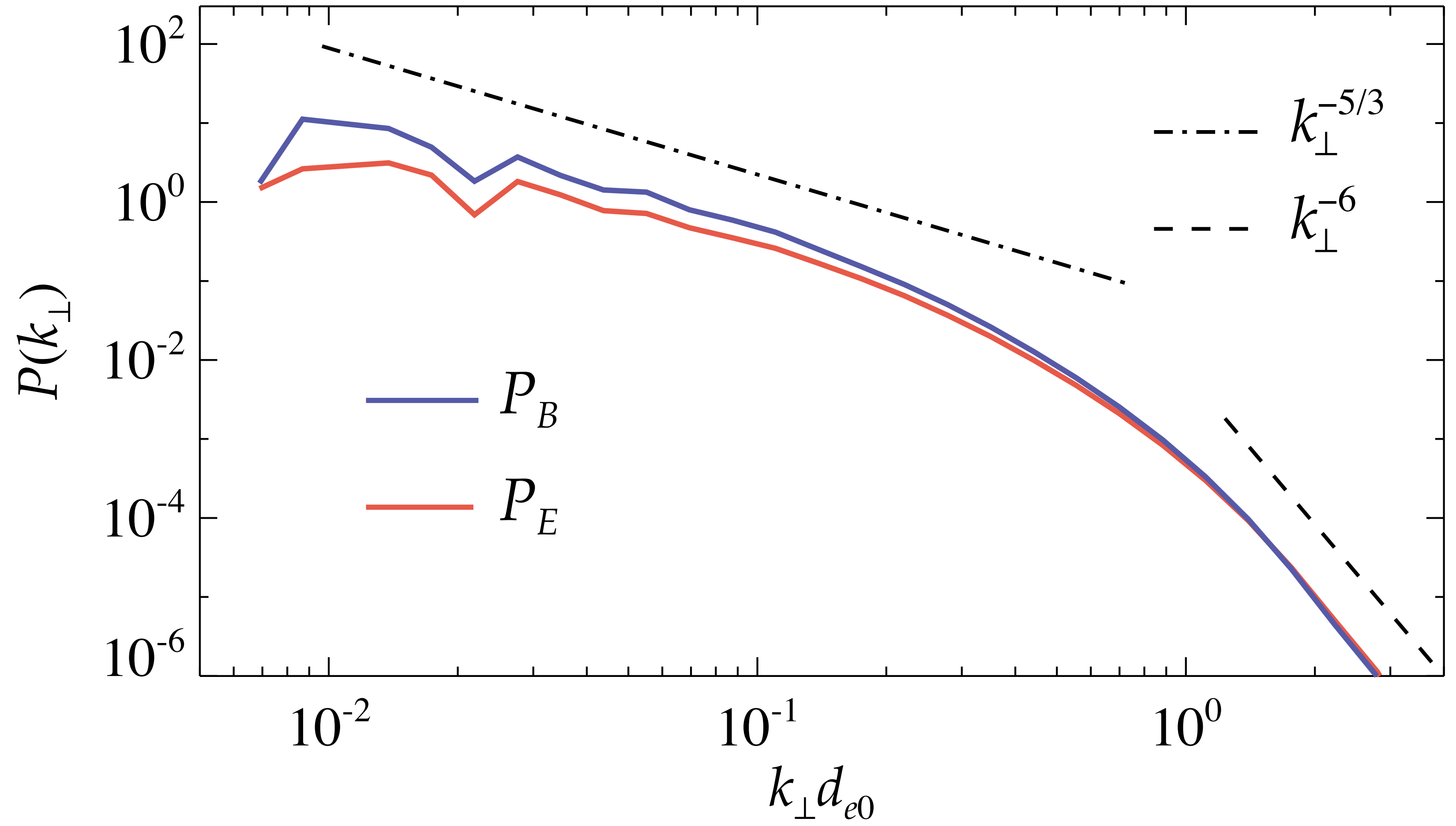
Comisso et al. 2020

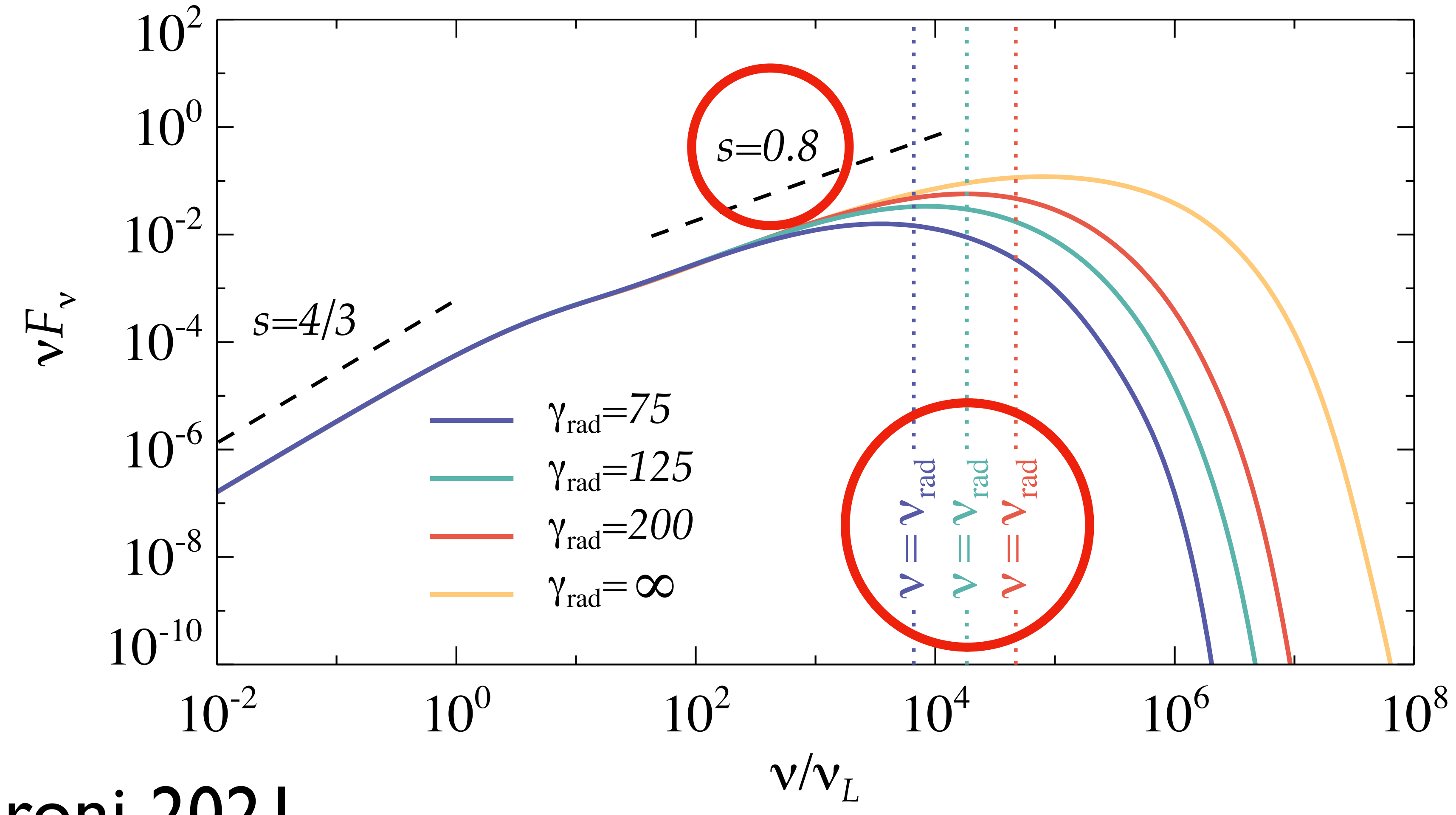
Tackling turbulence in the fast synchrotron cooling regime

$$\mathbf{F}_{RR} = \frac{2}{3} r_0^2 \left[(\mathbf{E} + \beta \times \mathbf{B}) \times \mathbf{B} + (\beta \cdot \mathbf{E})\mathbf{E} \right] - \frac{2}{3} r_0^2 \gamma^2 \beta \left[(\mathbf{E} + \beta \times \mathbf{B})^2 - (\beta \cdot \mathbf{E})^2 \right]$$



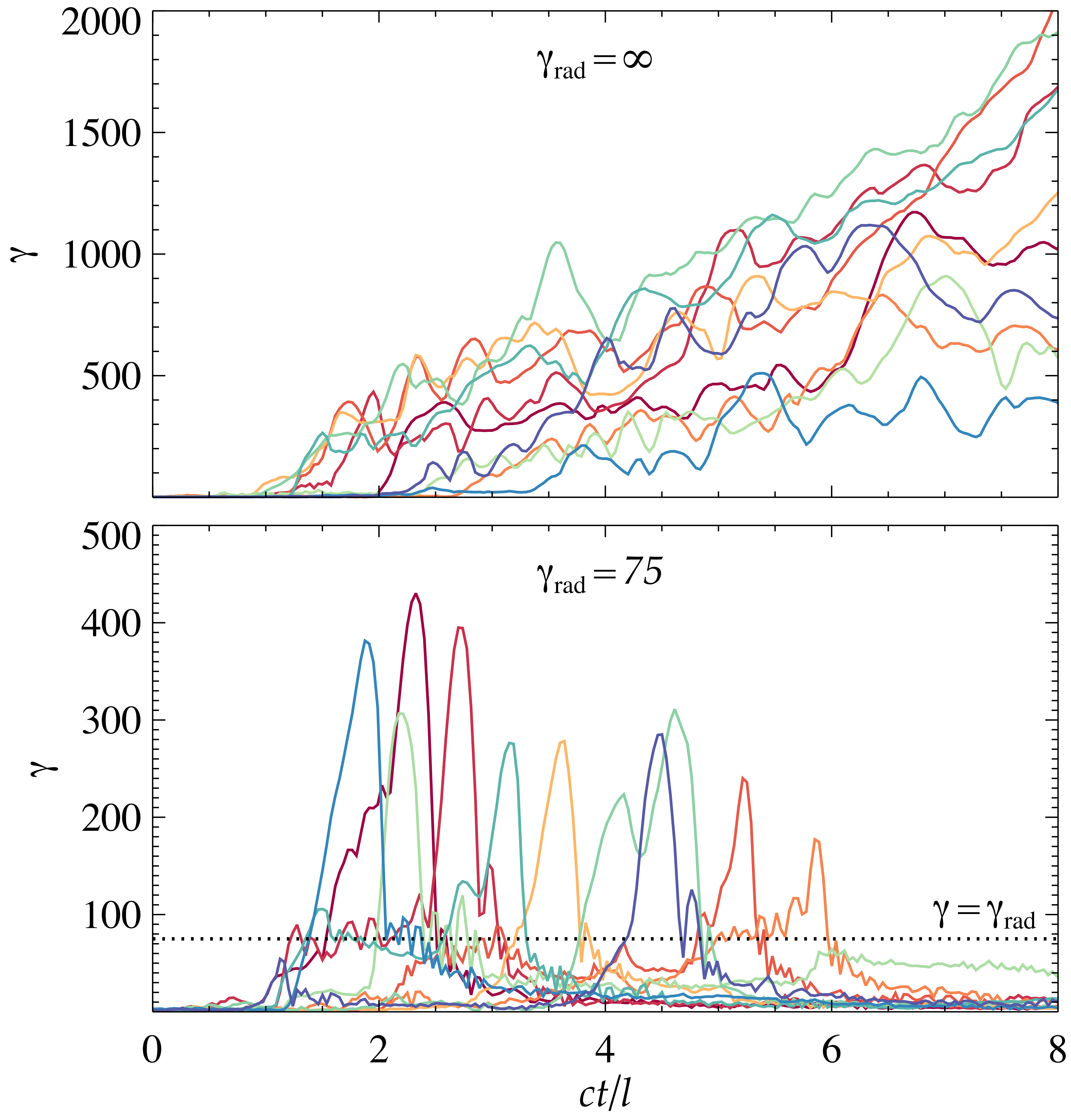
PIC simulations with 3072^3 cells
$(L/d_e = 1024)$



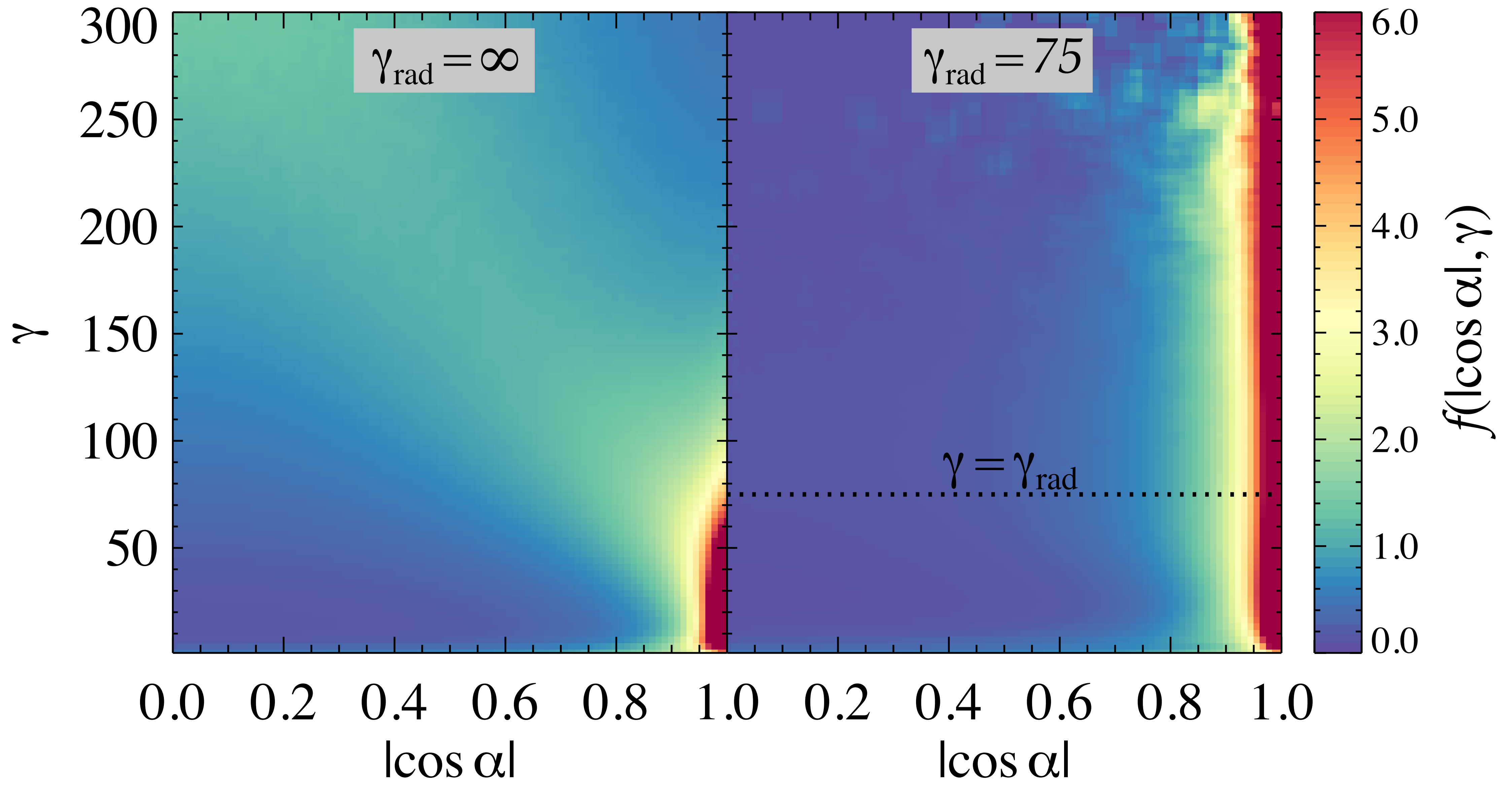


Comisso and Sironi 2021

Extreme particle acceleration and abrupt cooling



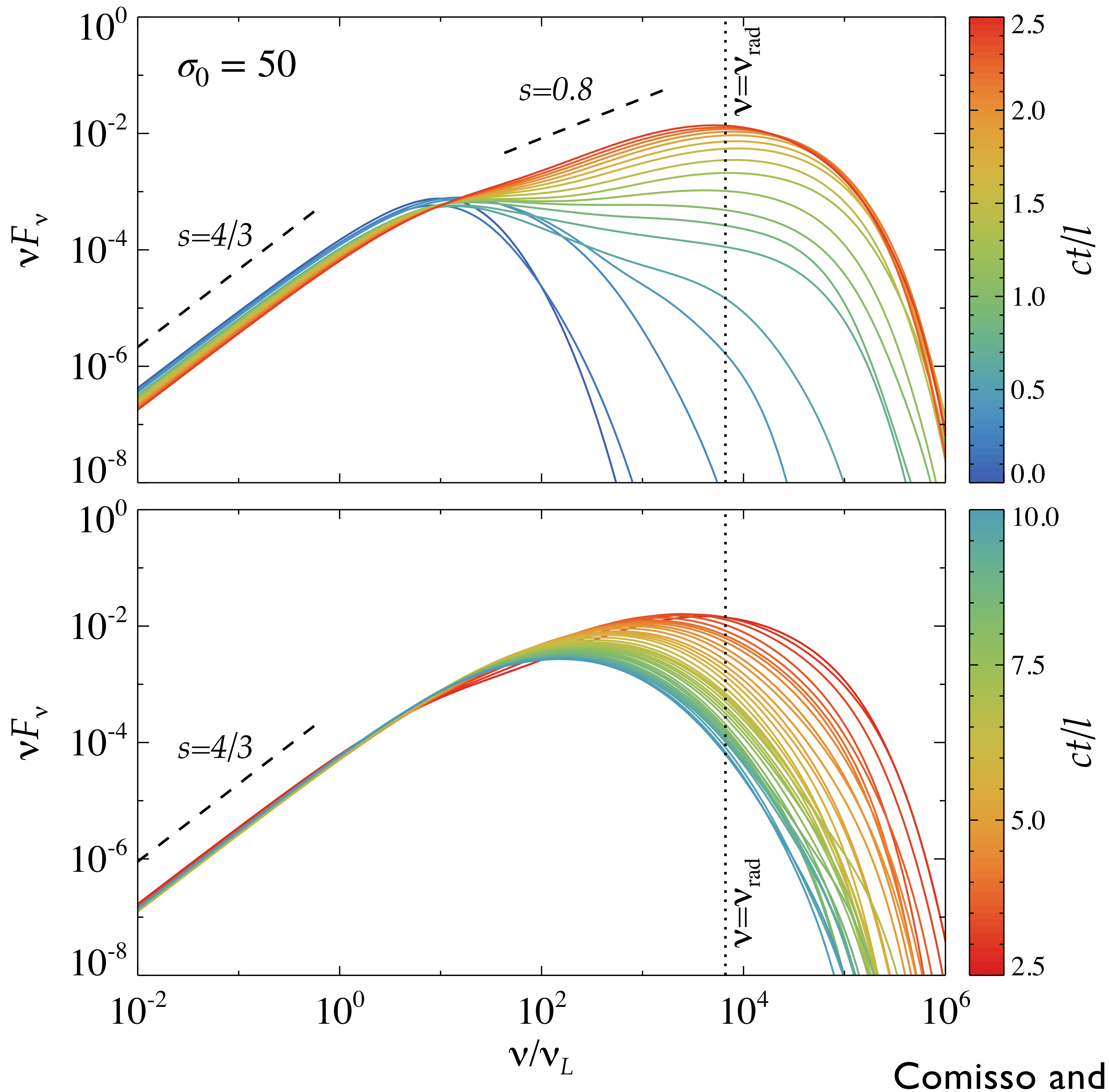
(see Nättilä & Beloborodov 2021 for IC)



- Particles with $\gamma \gg \gamma_{rad}$ thanks to $|\cos \alpha| \sim 0$

- Energy radiate away in fraction of gyroperiod: $\tau_{cool}(\gamma \gg \gamma_{rad}) \ll \gamma_{rad}/(\omega_L E/B)$ when $\sin \alpha \sim 1$

Radiative turbulence produces hard synchrotron spectra



Comisso and Sironi 2021

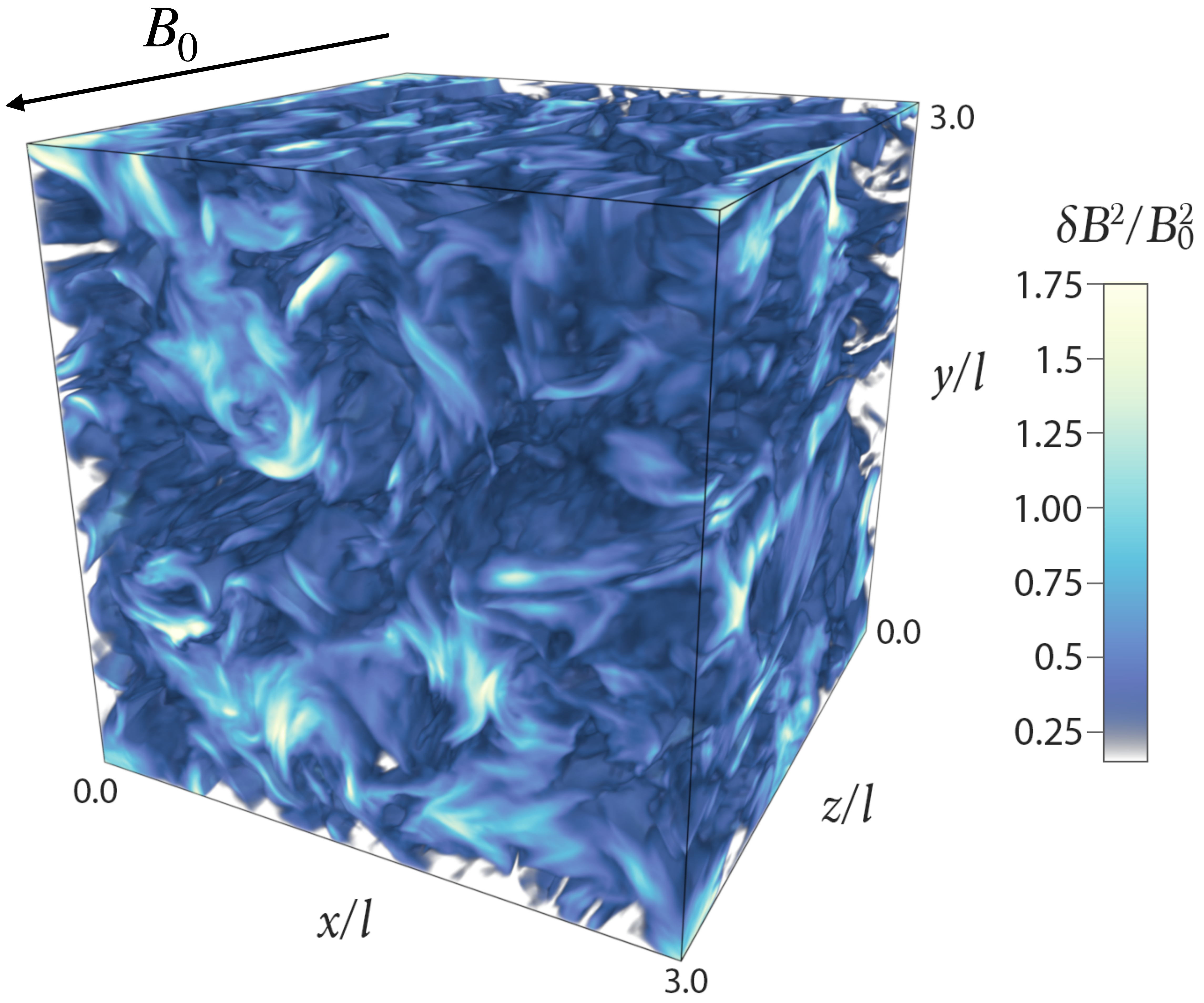
1. Energy carried away by radiation within a few eddy turnover times l/c
2. Excess of synchrotron radiation above $\nu_{\rm rad}$ (as high as $35\,\%$)
3. Hard synchrotron spectrum $\nu F_\nu \propto \nu^s$ (exceeds the limit $s = 1/2$)

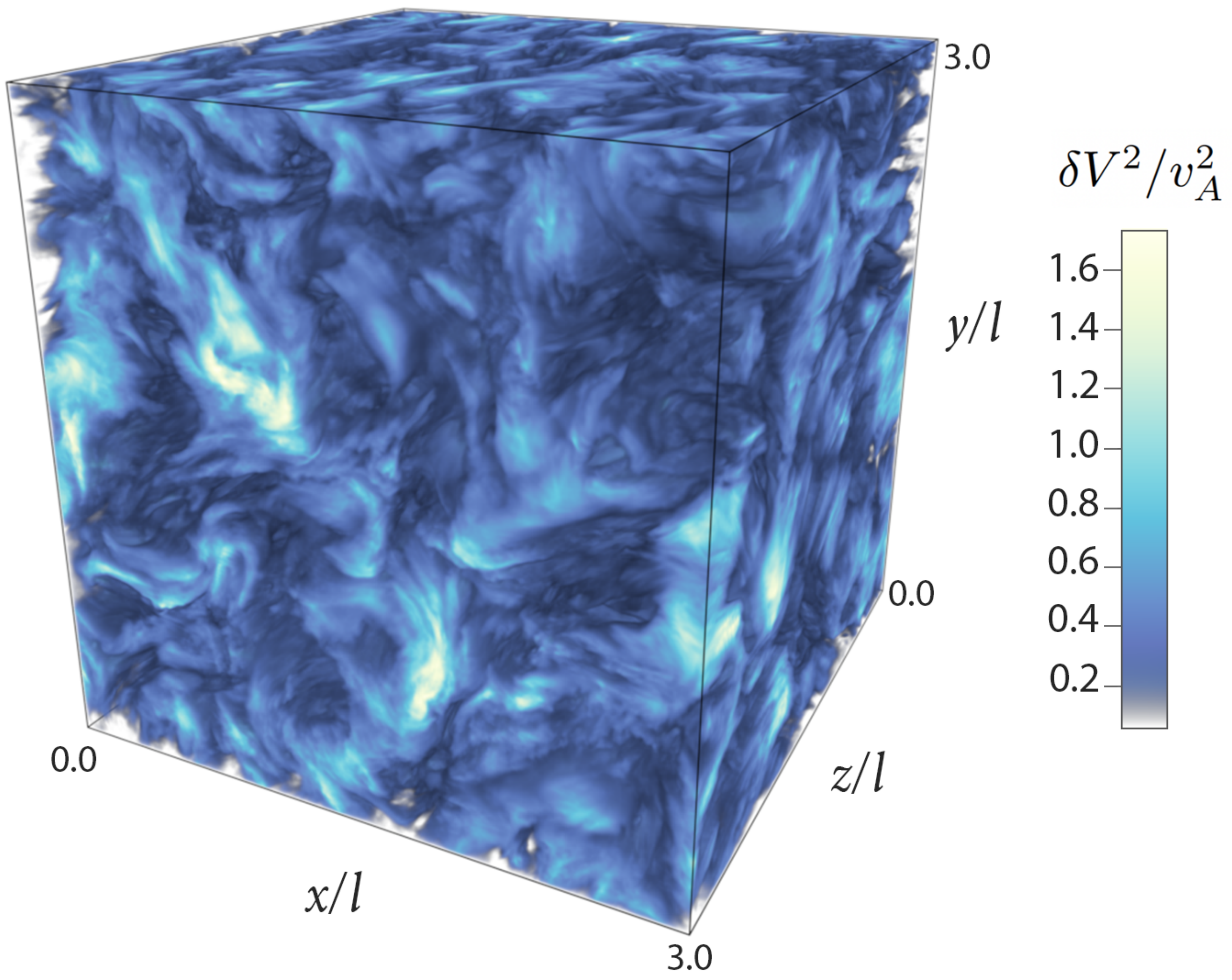
   - 2 & 3 are due to acceleration with $|\cos\alpha| \sim 0$ thanks to reconnection

✓ Radiative relativistic turbulence can potentially explain hard νF_ν spectra

Nonrelativistic turbulence in highly magnetized (low β) plasmas

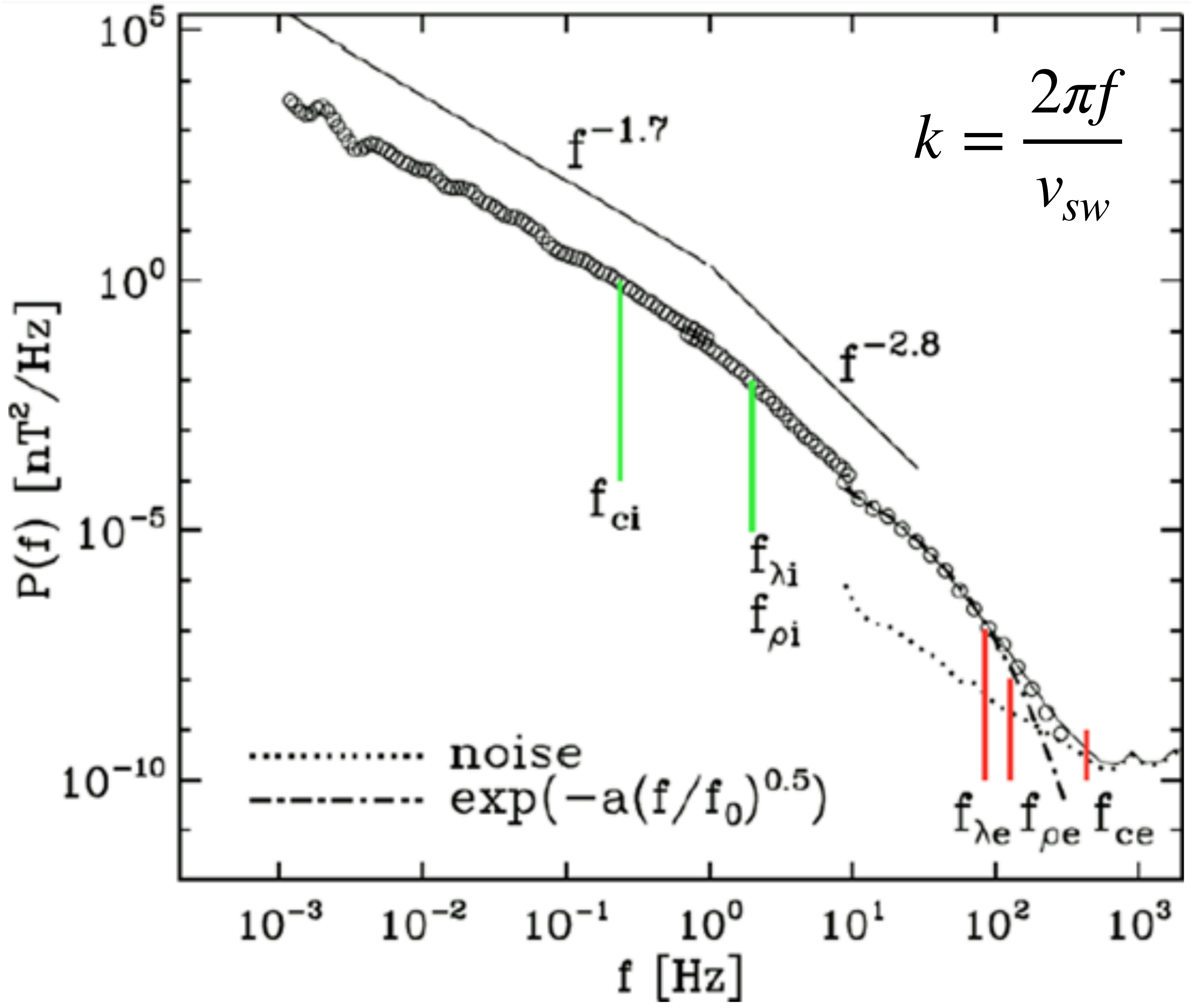
$$\delta B/B_0 = 1\ ,\ \beta = 0.08\ ,\ L^3 = (60 d_i)^3\ ,\ m_i/m_e = 50$$





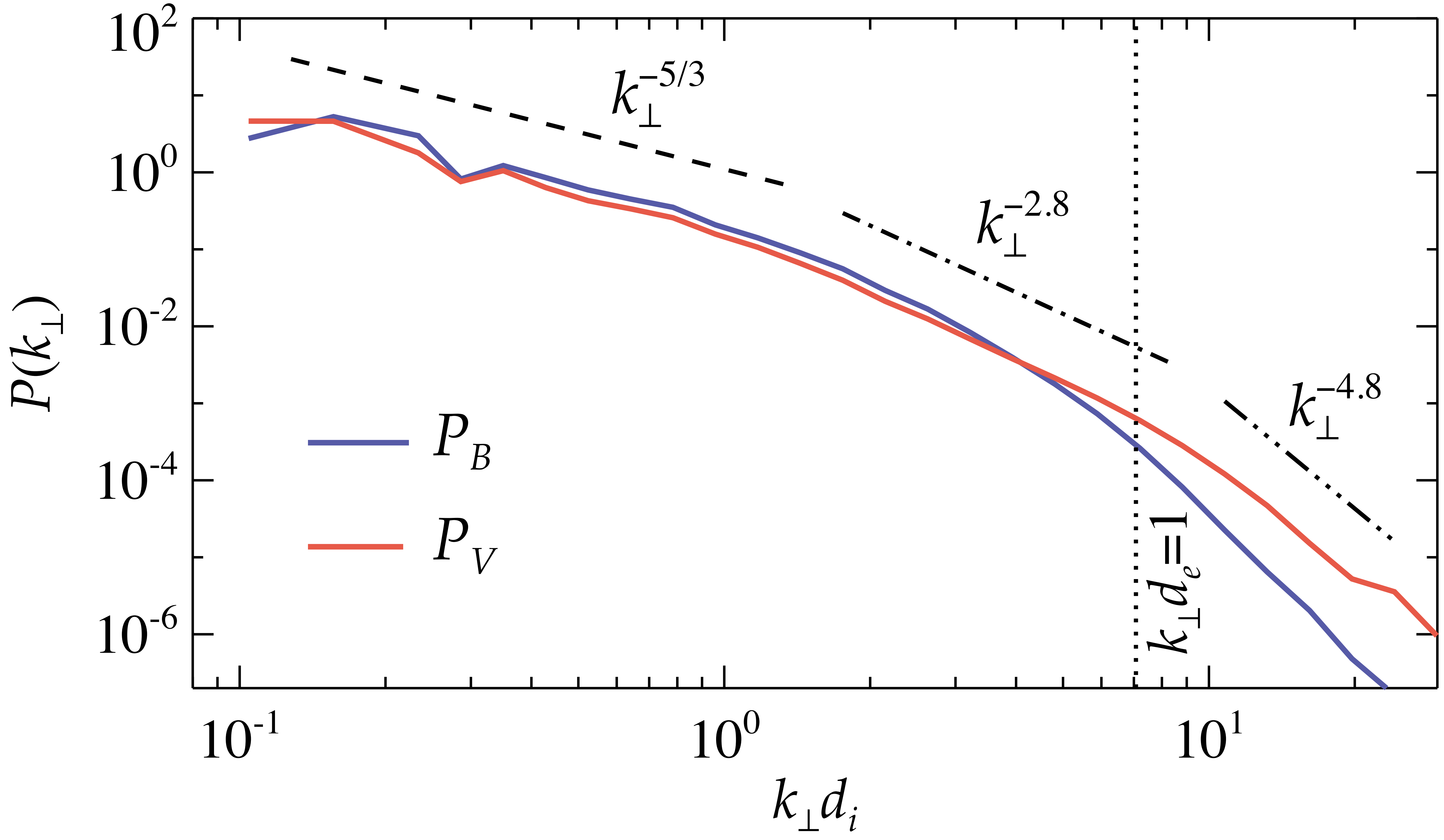
Nonrelativistic turbulence in highly magnetized (low β) plasmas

Magnetic power spectrum of Solar Wind



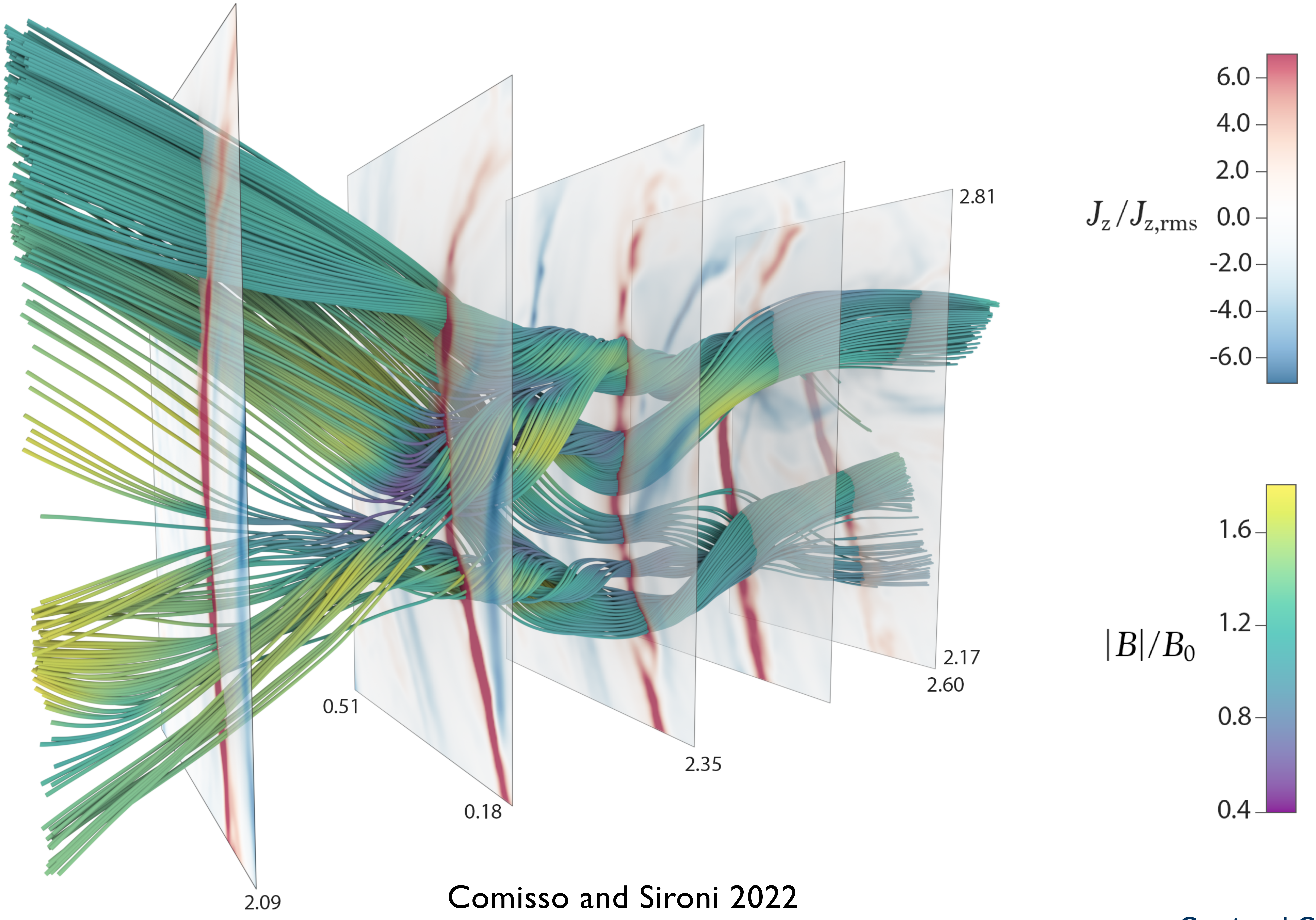
Alexandrova et al. 2013

Power spectra from low-β PIC simulation



Comisso and Sironi 2022

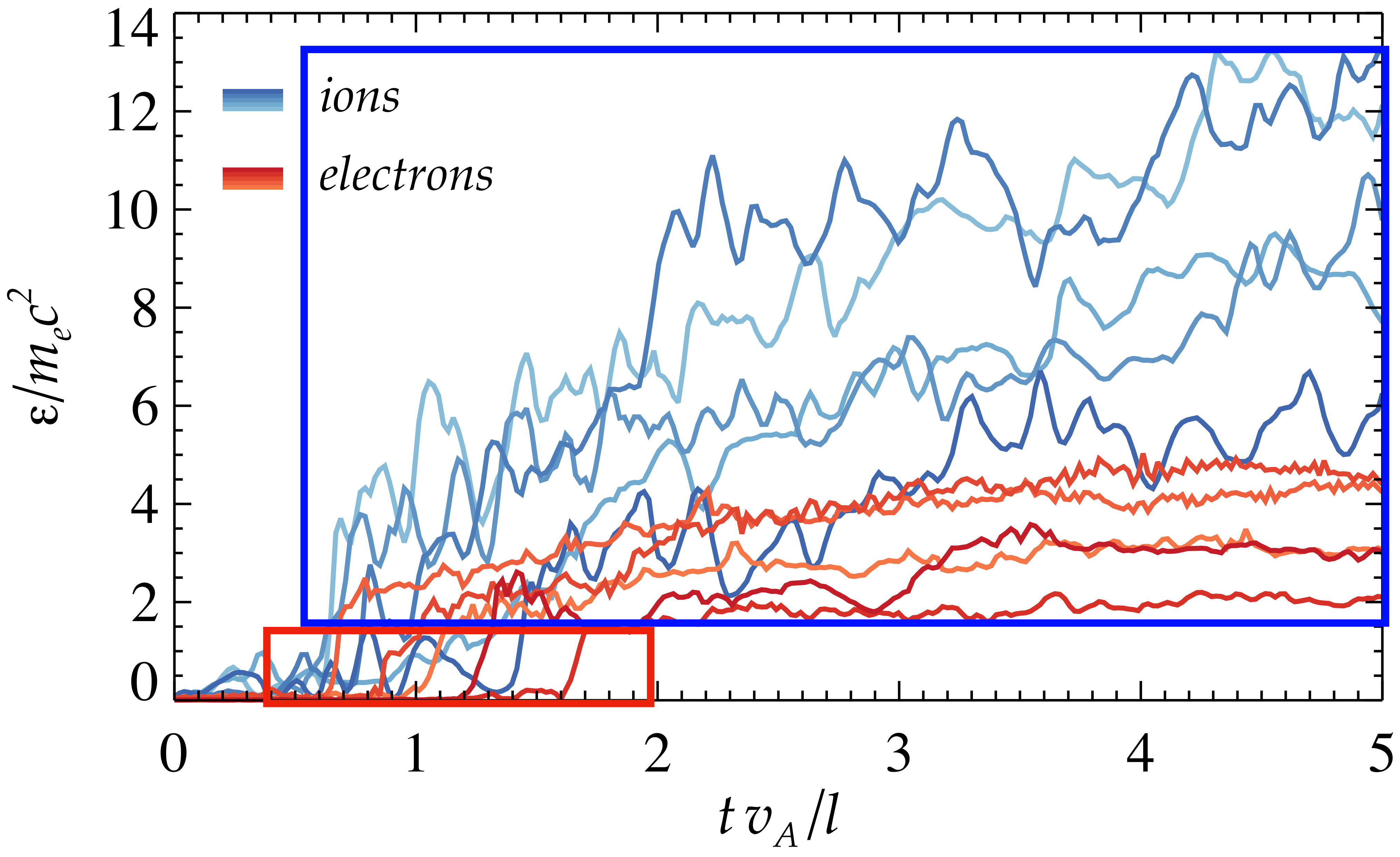
Magnetic reconnection occurring within the turbulent cascade



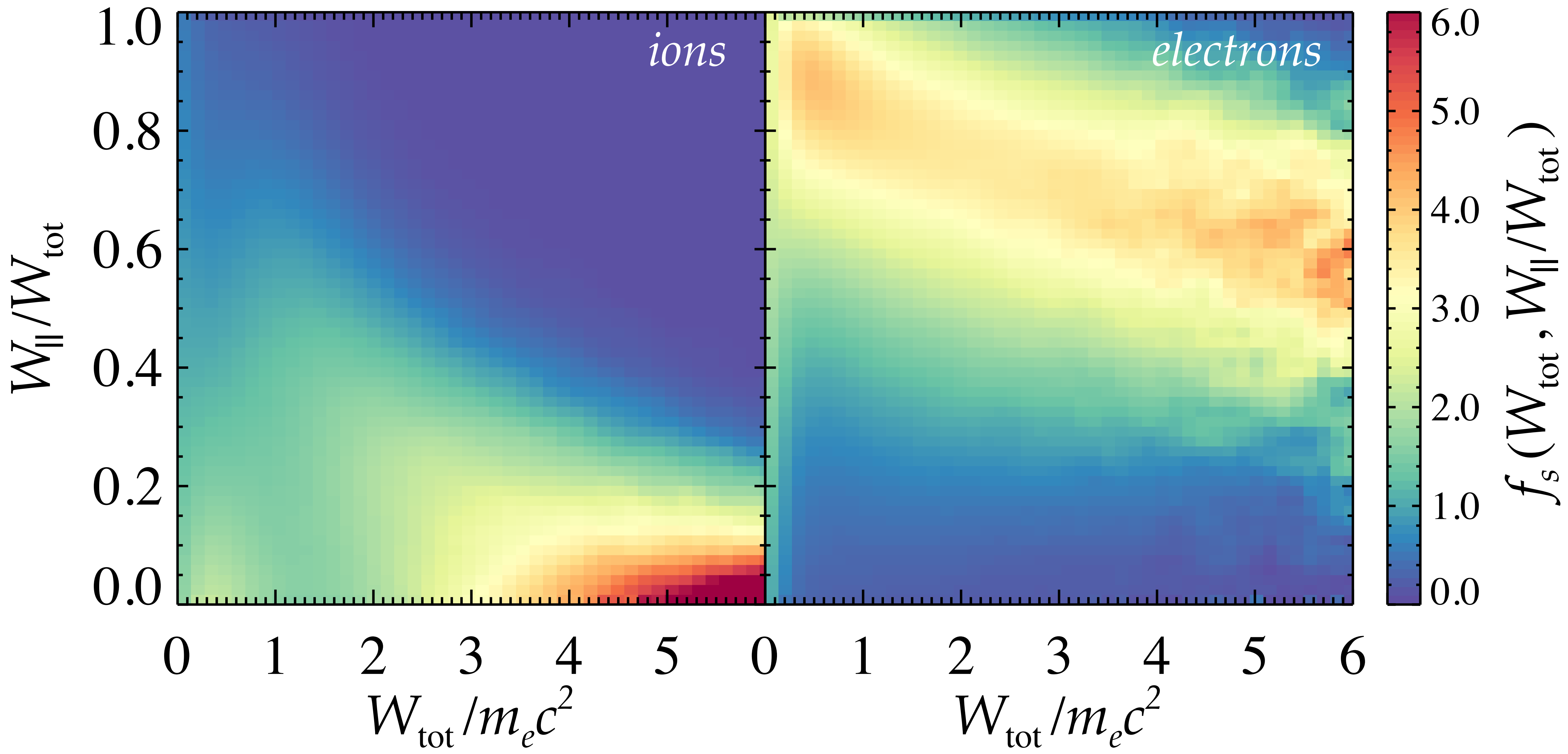
Comisso and Sironi 2022

Energization of ions and electrons

$$\varepsilon = (\gamma - 1) m_s c^2$$



$$W_{\|,\perp}(t) = q \int_0^t \mathbf{E}_{\|,\perp}(t') \cdot \mathbf{v}(t')\, dt'$$



- Two-stage acceleration process:

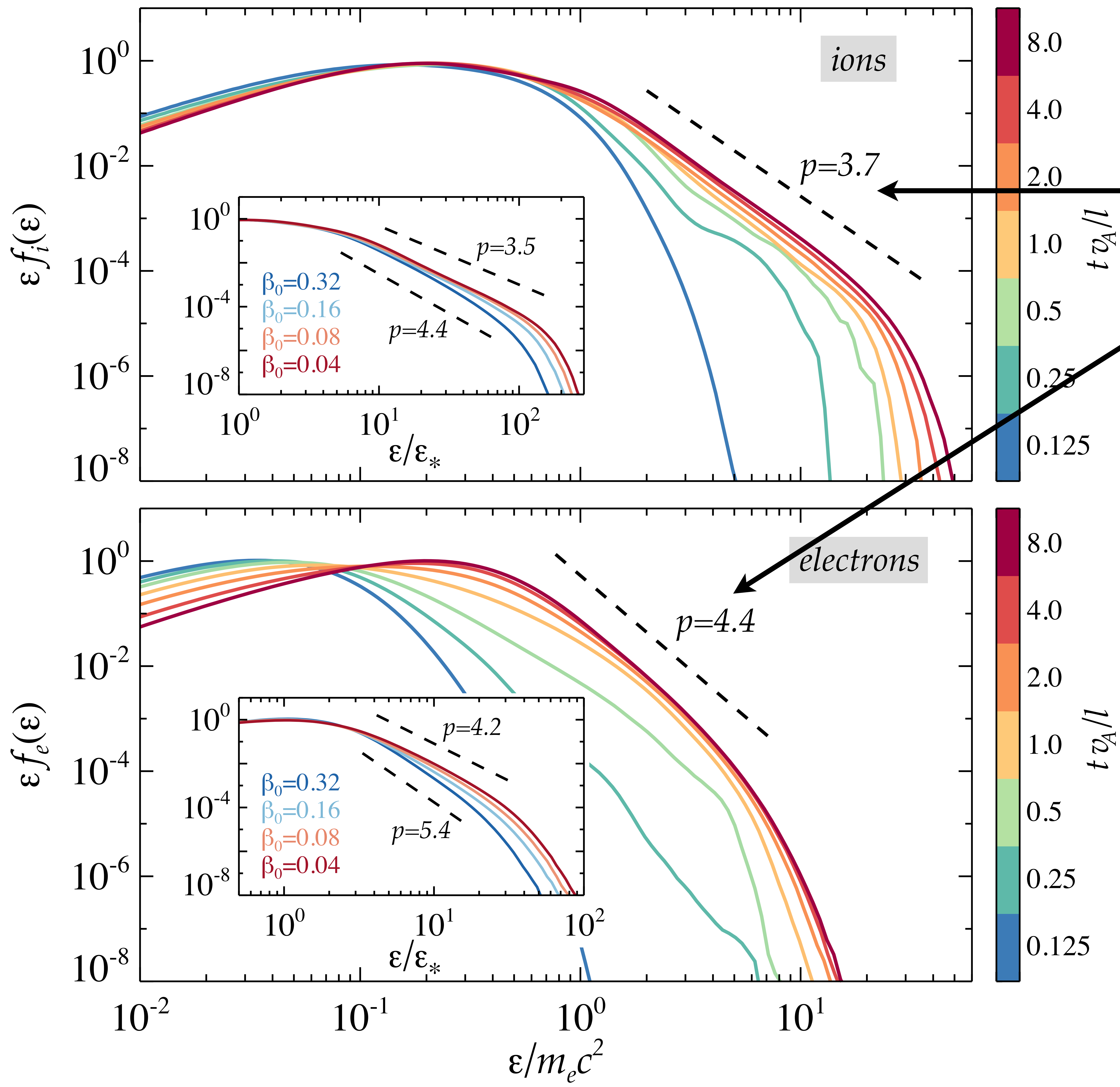
  (1) particle injection

  (2) stochastic Fermi acceleration

$f_{i,e}(W_{\rm tot}, W_{\|}/W_{\rm tot})$ from tracked particles

- Ions gain energy almost entirely via $E_{\perp}$
- Electrons acceleration is initiated by $E_{\|}$

Self-consistent development of nonthermal power-law tails



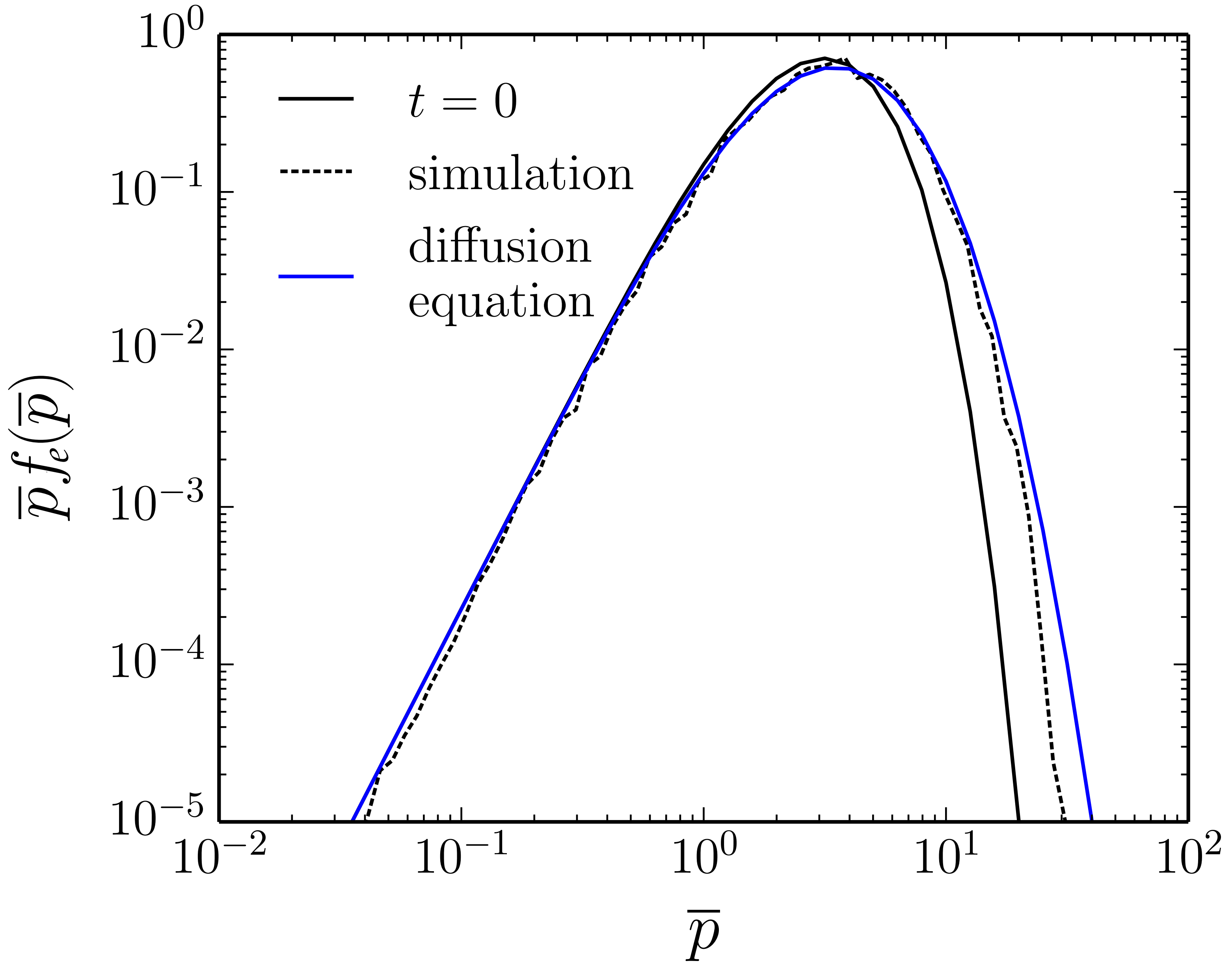
Comisso and Sironi 2022

Power-law tails develop *self-consistently*

$$f_s(\varepsilon)\,d\varepsilon \propto \varepsilon^{-p} d\varepsilon \qquad (s = i, e)$$

$$\varepsilon = (\gamma - 1)m_s c^2$$

Test-Particles in MHD fields failed to produce power laws



Lynn+ 2012, '13, '14

A few key takeaways

- Fully Kinetic Simultaneous Treatment of Turbulence, Reconnection, and Particle Acceleration
- High-Energy Particles are Generated Self-Consistently as a By-Product of Turbulence + Reconnection
- Particle Acceleration Follows a Two-Stage Process
  - 1st stage - particle injection by magnetic reconnection
  - 2nd stage - nonresonant Fermi-like stochastic acceleration
- Turbulence + Reconnection Generate Anisotropic Pitch Angle Distributions
- Anisotropic Pitch Angle Distributions affect the Synchrotron Spectrum produced by the Energetic Particles