Relativistic Fermi acceleration

Martin Lemoine

Institut d’Astrophysique de Paris

CNRS – Sorbonne Université

1. Relativistic Fermi acceleration: general remarks
2. Application to particle acceleration in turbulent flows
3. Some consequences and some open questions for phenomenology

Collaborators:

Virginia Bresci	(graduate student at IAP)
Camilia Demidem	(PhD 2019, NORDITA, Sweden)
Arno Vanthieghem	(PhD 2019, SLAC/Stanford)
Laurent Gremillet	(CEA, France)
Guy Pelletier	(IPAG, France)

+ A. Bykov (Ioffe), M. Malkov (UCSD), L. Comisso (Columbia), L. Sironi (Columbia)

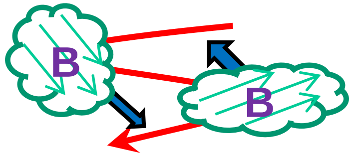
Microphysics of particle acceleration in the high-energy Universe

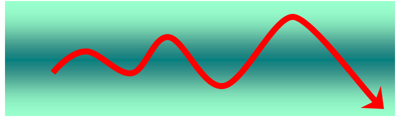
→ Lorentz force: $\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$ … what is the origin of ***E***?

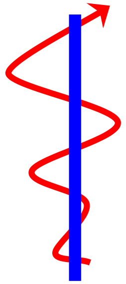
1. Acceleration à la Fermi: **highly conducting plasma…**

→ **large scale physics** (↔ very high energies?): corresponds to ideal Ohm's law $\boldsymbol{E} = -\boldsymbol{v}_p \times \boldsymbol{B}/c$…

→ **Fermi-type scenarios: magnetized turbulence, shear flows, shock waves**



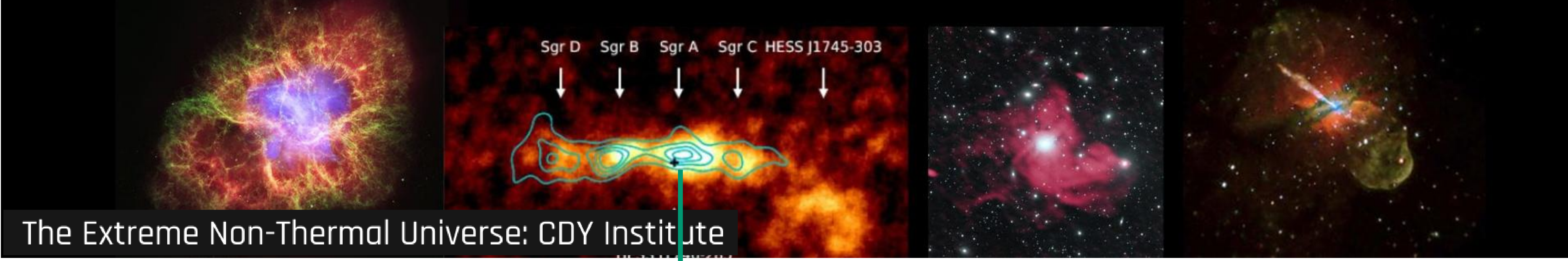


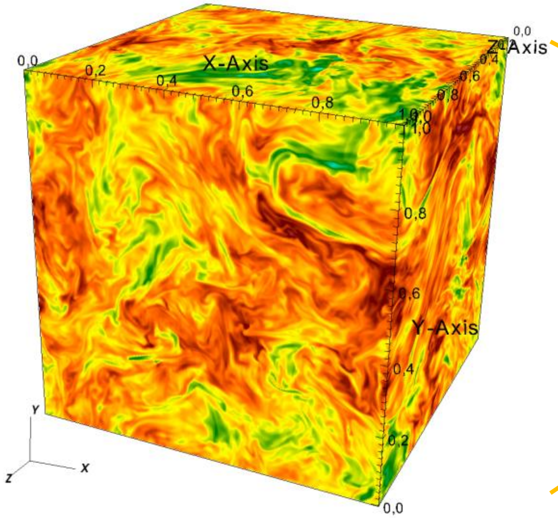


2. "Linear" accelerators: **non-MHD flows:** $\exists\, \boldsymbol{E}_{\parallel}$

→ acceleration can proceed unbounded along **E** (or at least **E**$_{\parallel}$)…

→ **gaps in magnetospheres, reconnection (on small scales)**





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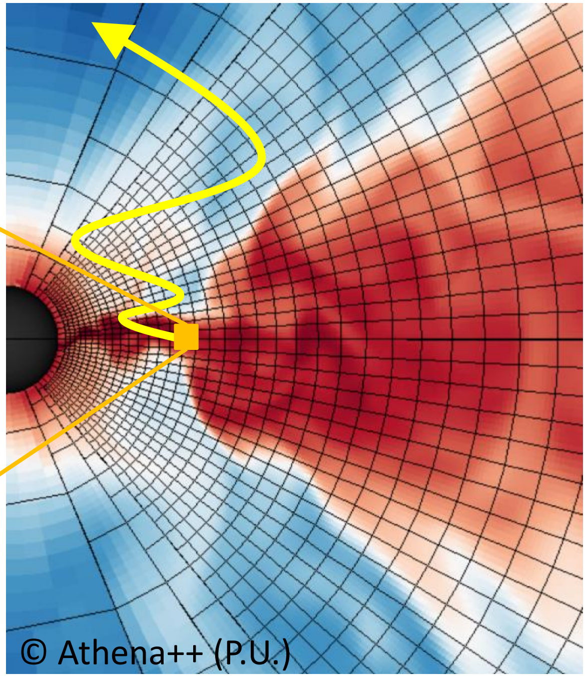

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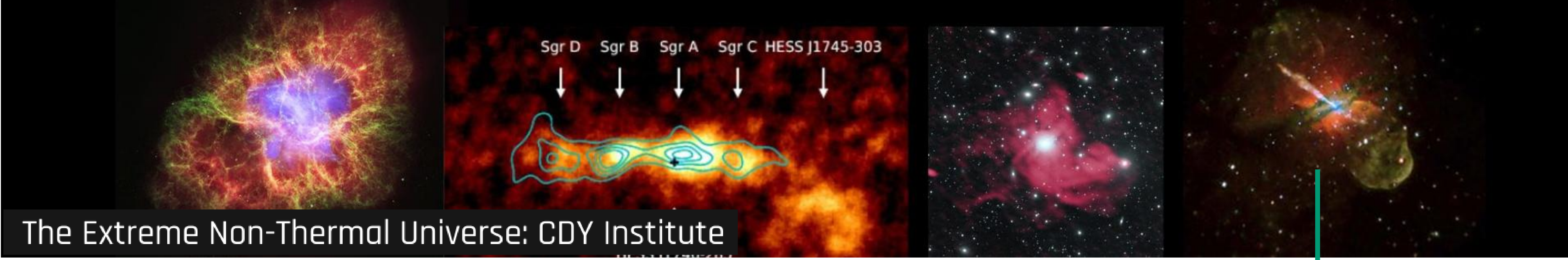
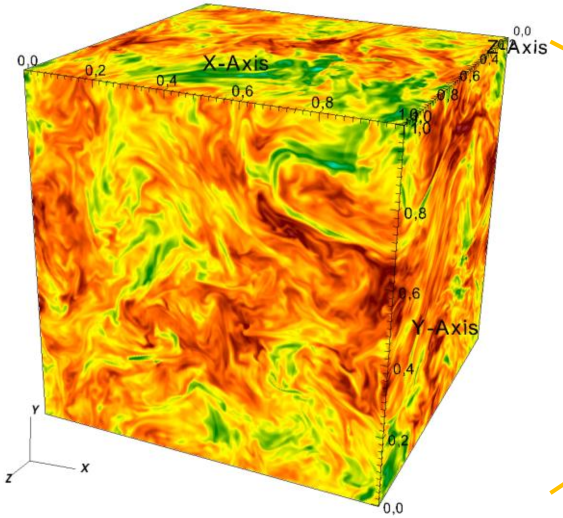
→ particles interact with a sheared, relativistic turbulent flow on a broad range of scales…

→ particles of different energies experience different accelerator configurations: fine structure smeared out over gyroscale…

<u>NB:</u> as in many astrophysical sources, a huge hierarchy between macroscopic scales (l_c turbulence scale, r_g) and microscopic scales (r_L): $r_g/r_L \sim 10^6$ for a GeV electron in 1G field… a challenge for numerical simulations!





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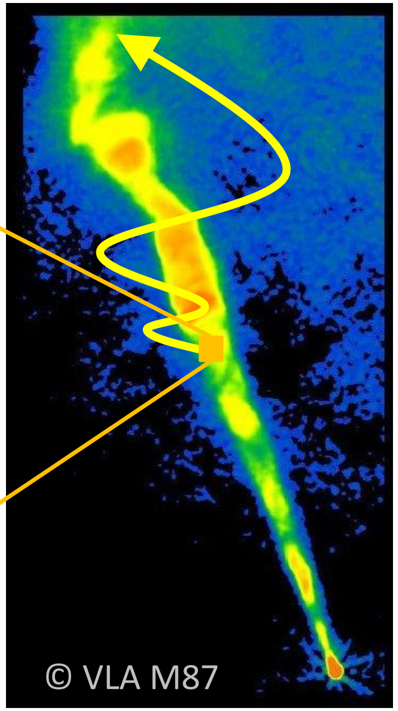

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→ particles interact with a sheared, relativistic turbulent flow on a broad range of scales…

→ particles of different energies experience different accelerator configurations: fine structure smeared out over gyroscale…

→ multi-stage, hierarchical acceleration scenarios, from non-ideal processes at the smallest length scales to Fermi-type processes at the highest energies

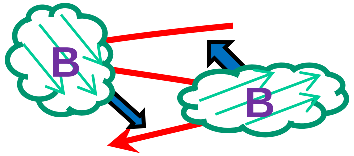
Microphysics of particle acceleration in the high-energy Universe

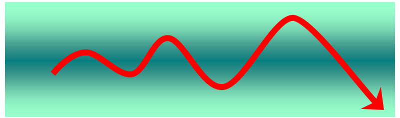
→ Lorentz force: $\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$ … what is the origin of ***E***?

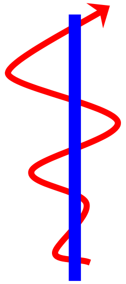
1. Acceleration à la Fermi: **highly conducting plasma…**

→ **large scale physics** (↔ very high energies?): corresponds to ideal Ohm's law $\boldsymbol{E} = - \boldsymbol{v}_E \times \boldsymbol{B} /c$…

→ **Fermi-type scenarios: magnetized turbulence,** **shear flows,** **shock waves**







→ **two essential characteristics:**

1. ***E*** vanishes in local frame: classification of Fermi scenarios according to geometry of ***E*** fields

2. scattering is essential to explore ***E*** fields through cross-***B*** transport

Fermi acceleration: the issue of scattering

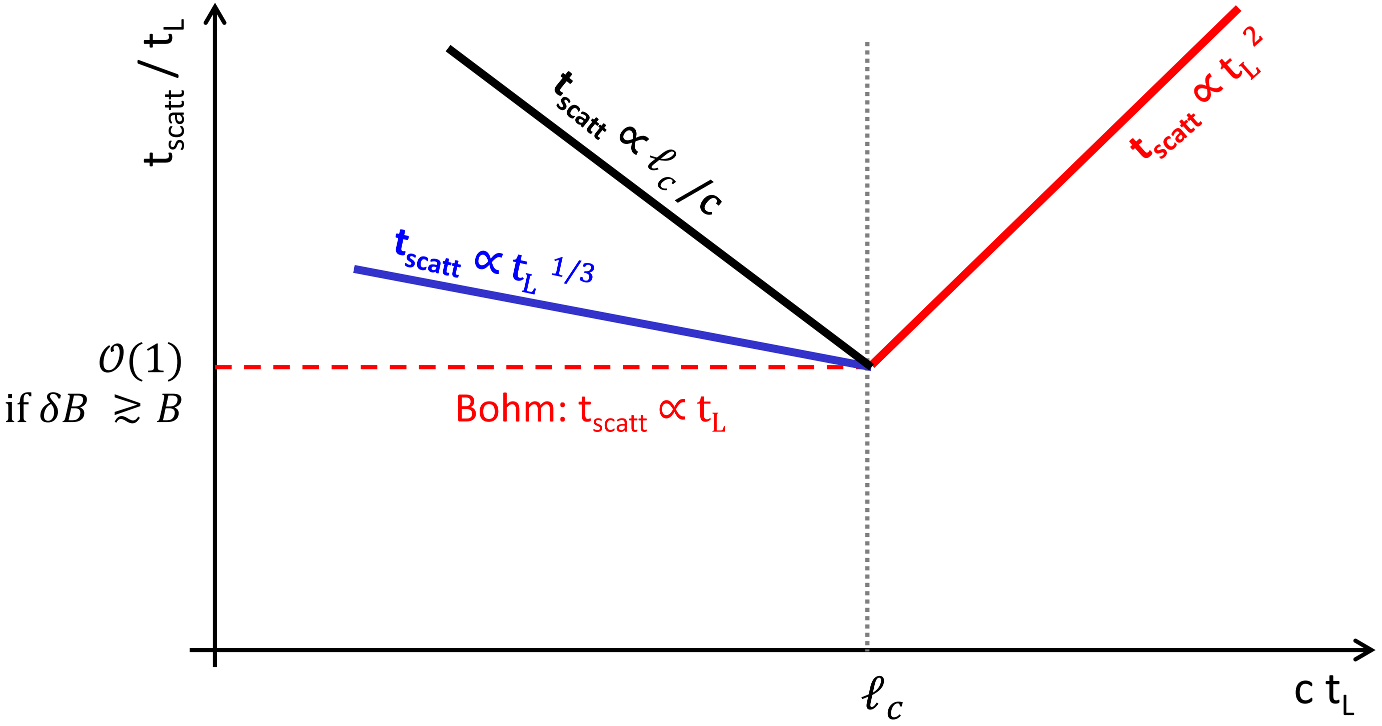
→ generic scaling: $t_{\rm acc} \simeq \frac{t_{\rm scatt}}{\beta_E^2}$ (applies to original Fermi, shock, turbulence… here $\beta_E \lesssim 1$)

→ <u>scattering timescale $t_{\rm scatt}$</u>: time it takes to deflect the particle by an angle of the order of unity,

$$t_{\rm scatt} \sim t_{\rm L}^{\alpha} \left(\ell_c/c\right)^{1-\alpha}$$ (l_c coherence length scale of turbulence)

… in absence of specific information, assume (too often!): **α ~ 1 Bohm regime**

… however:



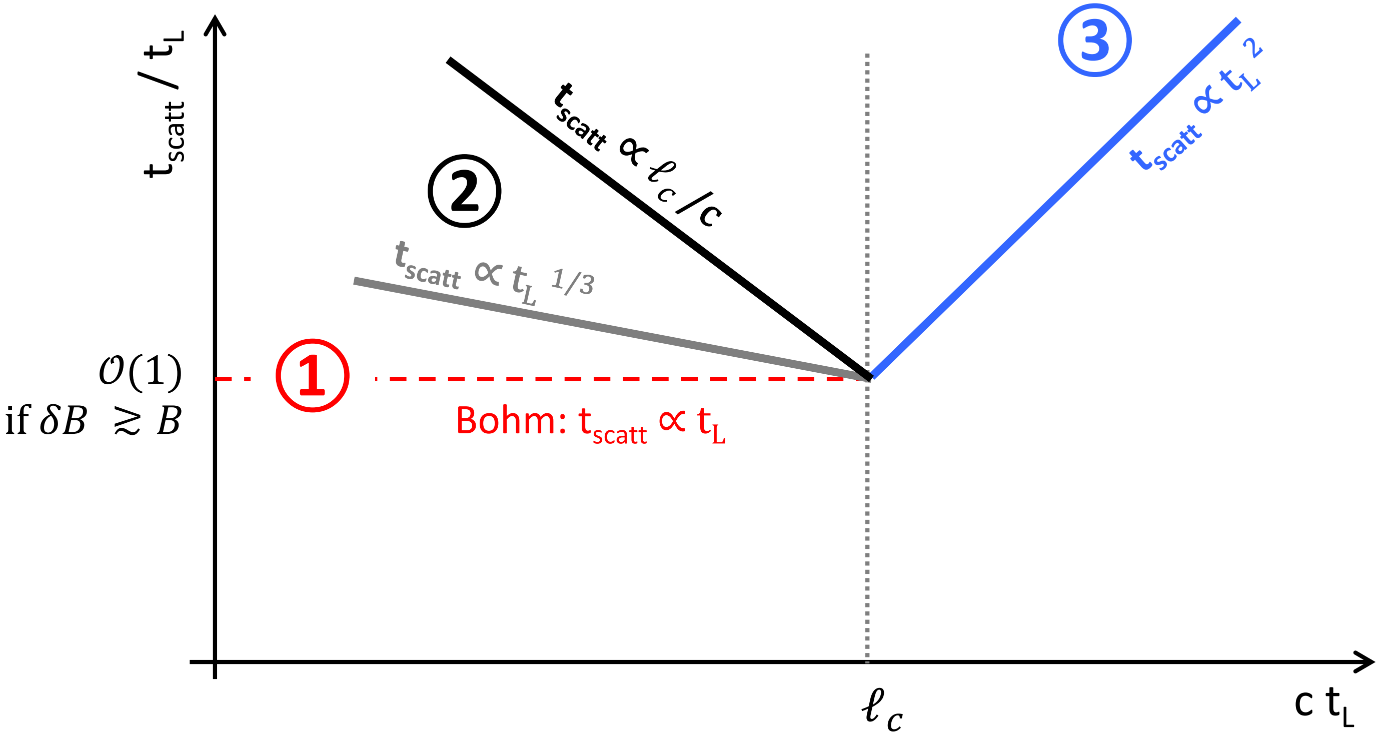
Fermi acceleration: the issue of scattering

① : extreme accelerator if $\beta_E \sim 1$, synchrotron from e at radiation reaction limit:

$$t_{\rm acc} \simeq \frac{t_{\rm scatt}}{\beta_E^2} \qquad \text{and} \qquad t_{\rm acc} \simeq \mathcal{A}\, t_{\rm L} \Rightarrow \epsilon_{\rm syn,max} \simeq \mathcal{A}^{-1}\frac{m_e c^2}{\alpha_{\rm e.m.}} \sim 100\,\mathcal{A}^{-1}\,{\rm MeV}$$

② : non-extreme for e, but can reach confinement (Hillas) limit for ions if $\beta_E \sim 1, \ell_c \sim$ source size, $\delta B \gtrsim B$

③ : particles decouple from turbulence at high-E, slow scattering…

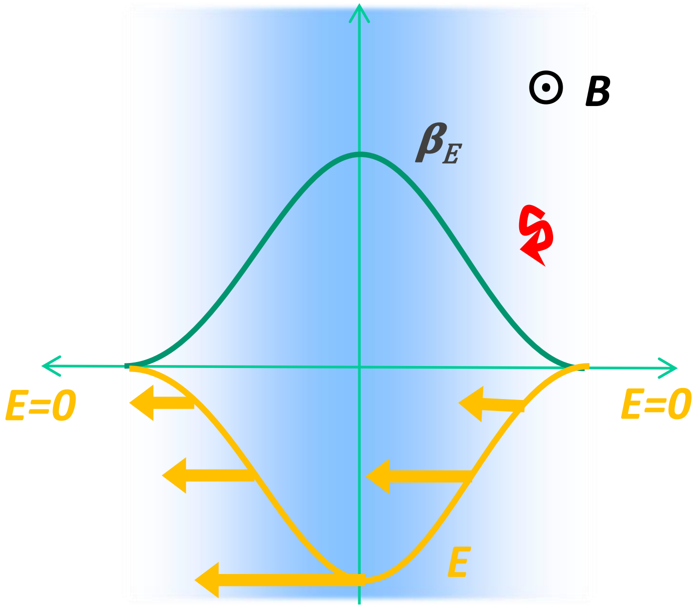


A case study: shear acceleration

→ in shear acceleration, particles gain energy by exploring varying (motional) electric field configurations[1]…

motional electric field: $\boldsymbol{E} = -\boldsymbol{\beta}_E \times \boldsymbol{B}$

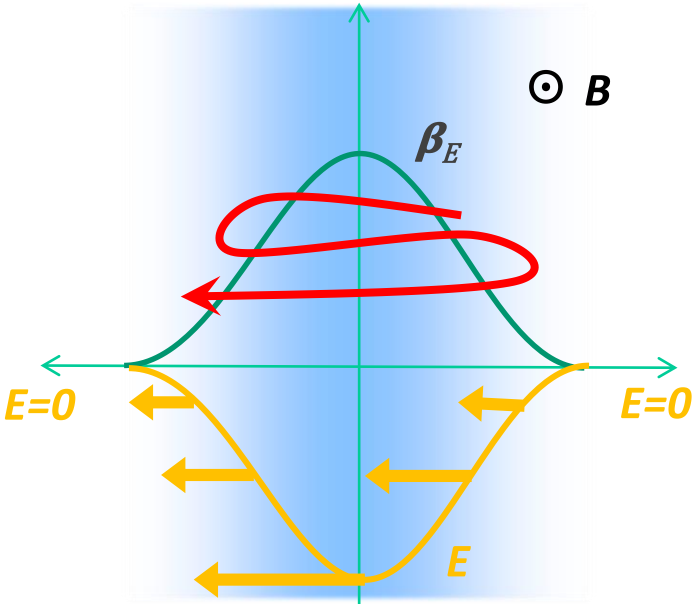
small mfp: $c\, t_{\mathrm{scatt}} \ll \beta_E/\nabla\beta_E$



particles with small mean free paths explore weak gradient of ***E***

⇒ slow acceleration…

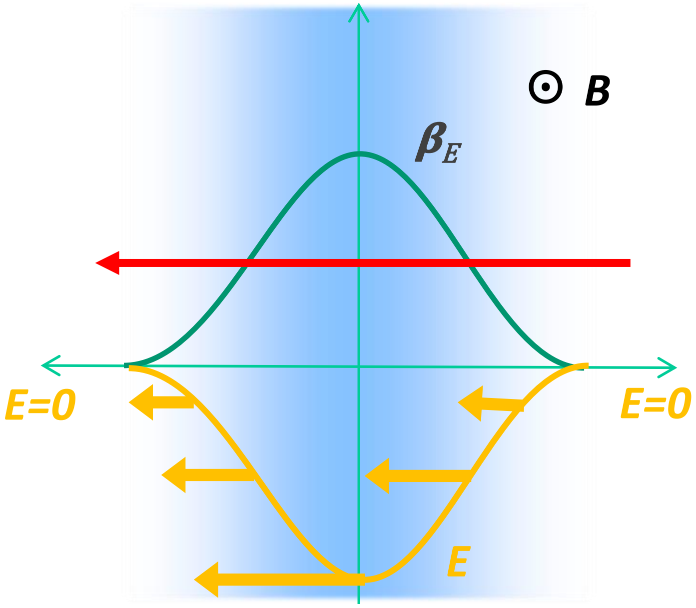
“resonant” mfp: $c\, t_{\mathrm{scatt}} \sim \beta_E/\nabla\beta_E$



particles with near resonant mean free paths explore strong gradient of ***E***

⇒ fast acceleration…

large mfp: $c\, t_{\mathrm{scatt}} \gg \beta_E/\nabla\beta_E$



particles with very large mean free paths do not see ***E****,* decouple from flow

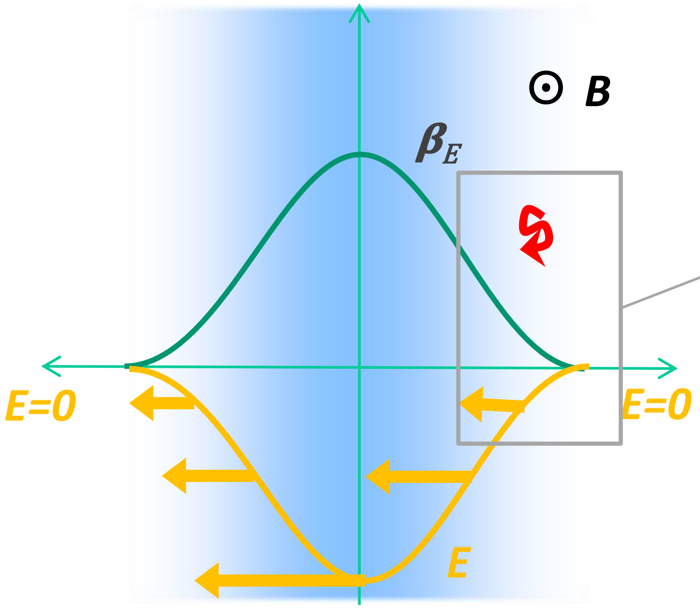
⇒ (very) slow acceleration…

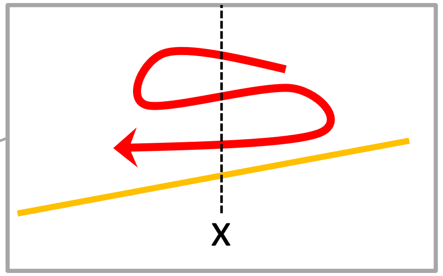
Refs: 1. e.g. Rieger 19

The role of the mean free path, and of velocity gradients

→ in shear acceleration, particles gain energy by exploring varying (motional) electric field configurations…
motional electric field: $\boldsymbol{E} = -\boldsymbol{\beta_E} \times \boldsymbol{B}$ … vanishes in frame moving at $\boldsymbol{\beta_E} = \boldsymbol{E} \times \boldsymbol{B}/B^2$

small mfp: $c\, t_{\rm scatt} \ll \beta_E/\nabla\beta_E$





… at x: in frame moving at β_E,
$\boldsymbol{E}(x)=0 \Rightarrow$ no acceleration…

… particle gains energy because of the existence of a gradient, which guarantees that the effect of ***E*** cannot be boosted away **everywhere**

*particles with small mean free paths explore weak gradient of **E***
⇒ slow acceleration…

… in shear acceleration (peculiar scaling!):

$$t_{\rm acc} \sim \frac{1}{{\beta_E}^2}\,\frac{\left(\beta_E/\nabla\beta_E\right)^2}{c^2 t_{\rm scatt}}$$

time it takes the particle to explore velocity gradient of length $\beta_E/\nabla\beta_E$ while traveling diffusively with step c $t_{\rm scatt}$

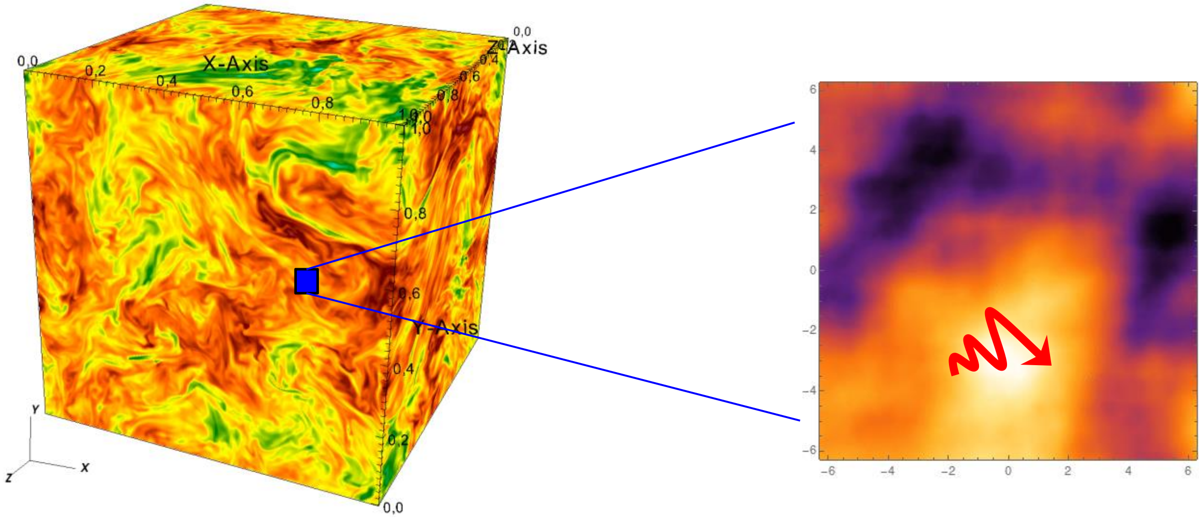
… Note: role of turbulence limited to scattering?

Stochastic Fermi acceleration in a large-scale, random flow

→ ***what matters is the shear of the velocity flow $\partial_\alpha {u_E}^\beta$***:

ideal MHD conditions: $\boldsymbol{E}$ vanishes in frame moving at $\boldsymbol{u_E} \propto \boldsymbol{E} \times \boldsymbol{B} \Rightarrow$ no acceleration in absence of shear

… $\partial_\alpha {u_E}^\beta \supset$ compression/dilation, shear, vorticity, + acceleration



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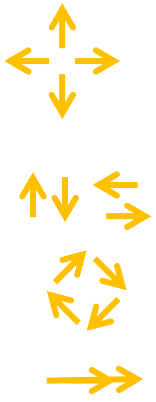


expansion/compression

shear

vorticity

acceleration/deceleration

→ ***can be seen as some generalization of discrete, point-like scattering of original Fermi, to continuous flow***

Refs: 1. Bykov+Toptygin 81, Ptuskin 88, …, Ohira 13, ML19, Demidem+20, ML21

Follow the particle momentum in the frame where E=0

→ convenient choice[1]: follow particle 4-velocity (γ', $\mathbf{u}'$) in (accelerated!) frame moving at $\boldsymbol{u_E}$

in that frame, no electric field…

⇒ Δ energy ∝ non-inertial forces characterized by velocity shear

→ approximation[2]:

$$\boxed{\frac{\mathrm{d}\gamma'}{\mathrm{d}\tau} = -\gamma' u'_{\parallel}\,\boldsymbol{a_E}\cdot\boldsymbol{b} - {u'_{\parallel}}^2\,\Theta_{\parallel} - \frac{1}{2}{u'_{\perp}}^2\Theta_{\perp}}$$

effective gravity along field line

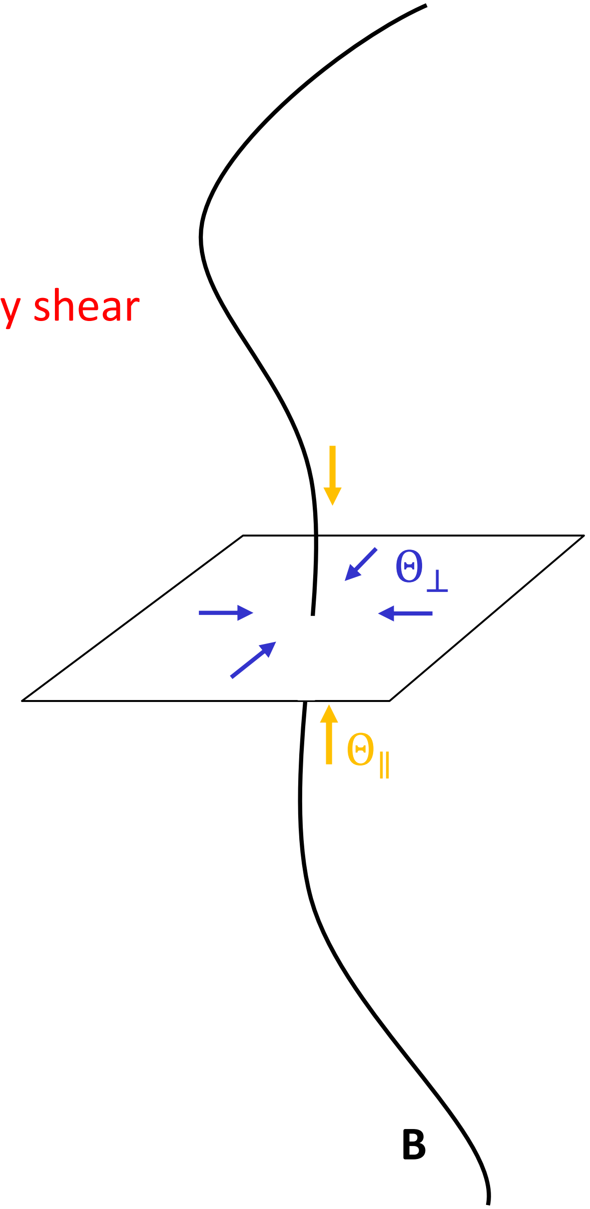
$$\boldsymbol{a_E} = u_E^{\alpha}\,\partial_\alpha\,\boldsymbol{u_E}$$

velocity shear along field line

$$\Theta_{\parallel} = b^{\alpha} b^{\beta}\partial_\alpha u_{E\,\beta}$$

compression transverse to field line

$$\Theta_{\perp} = \left(\eta^{\alpha\beta} - b^{\alpha} b^{\beta}\right)\partial_\alpha u_{E\,\beta}$$



[considers all scales ≫ r_L, ignores scales ≪ r_L , assumes local gyromotion around curved magnetic field]

Refs: 1. ML19 2. ML21

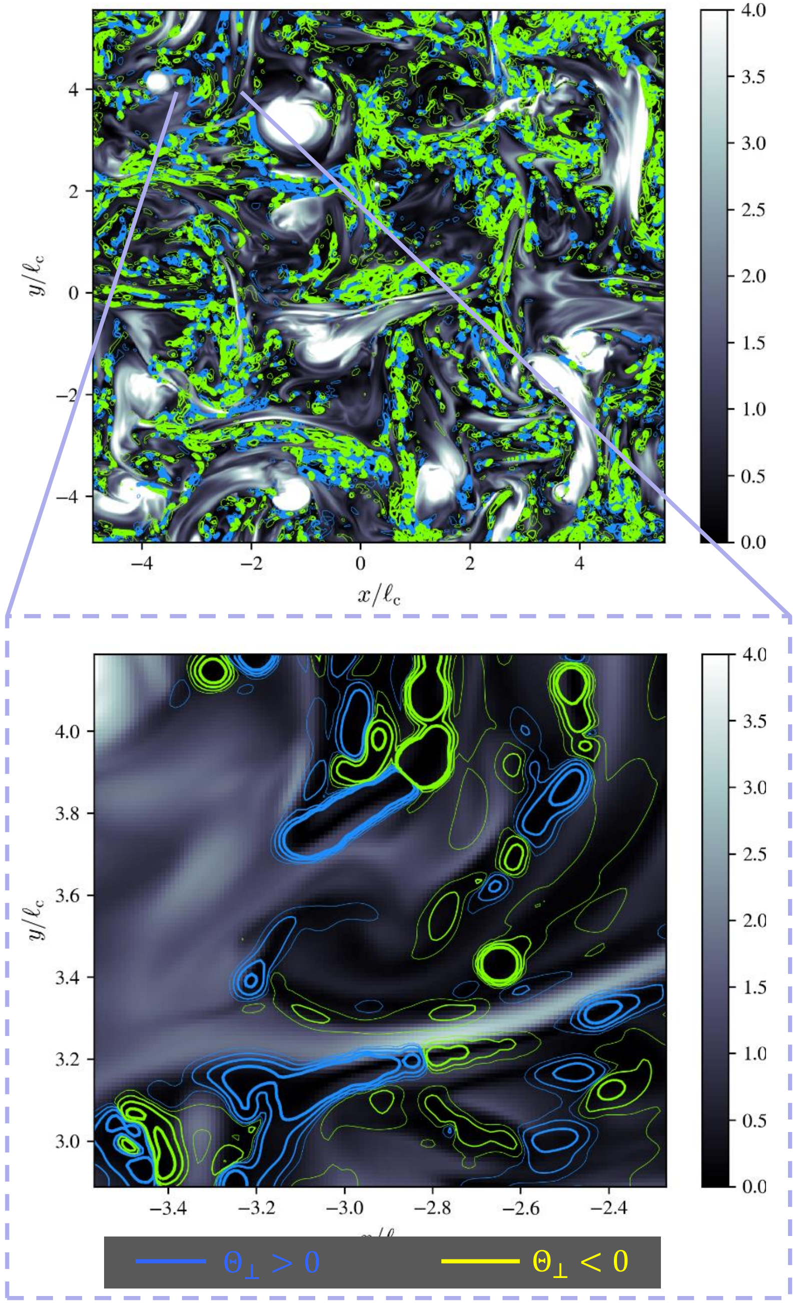
Acceleration in gradients of velocity field

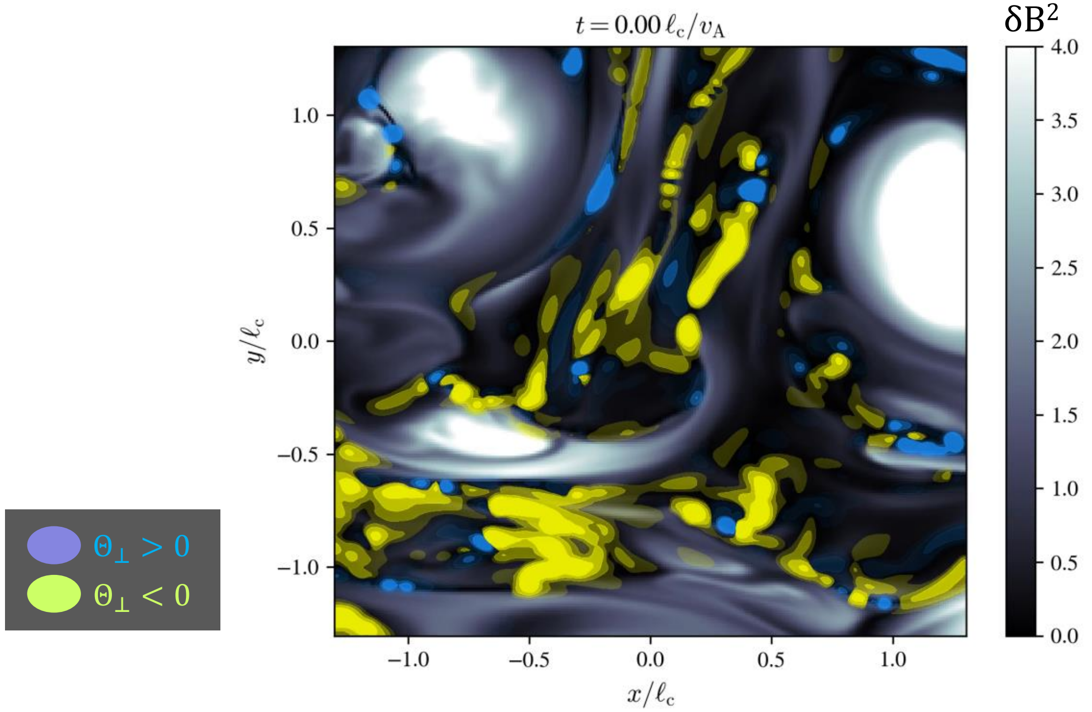
→ distinctive features:

acceleration scales with gradient of magnetic energy density (unlike QLT: magnetic energy density)

acceleration sites occupy only a small filling fraction of the total volume (unlike QLT: homogeneous statistics)

in each site, particle gains or loses energy according to sign of **Θ** (unlike Fermi: head-on vs tail-on)

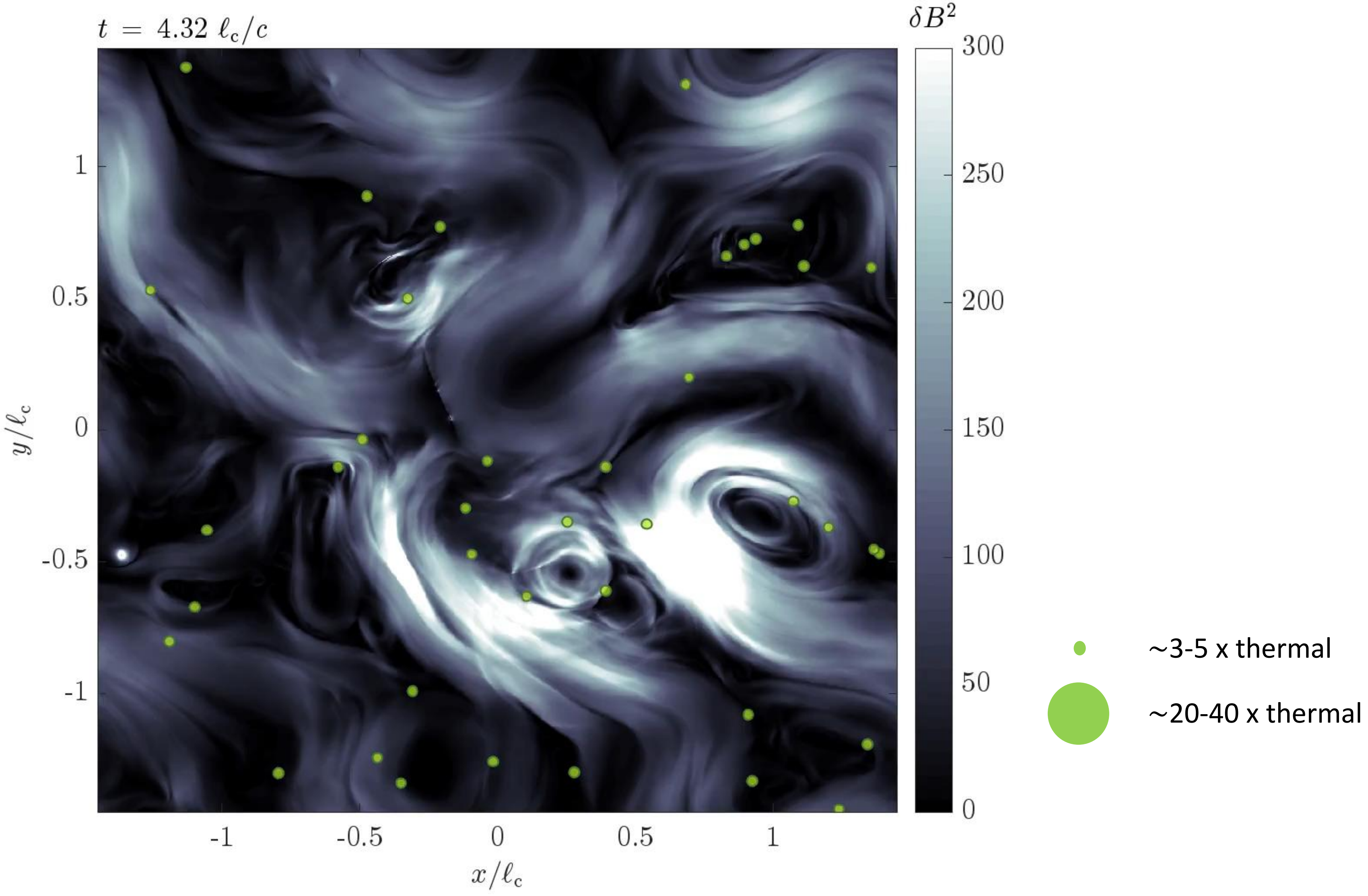




JHU-MHD database: 3D incompressible MHD, 1024^3, $v_A = 0.4$ c

View from particle-in-cell simulations

→ topology of acceleration sites: ∼ located in regions of gradients of magnetic energy



© V. Bresci, L. Gremillet, M. L.: 2D PIC, driven turb., e^+e^-, $10\,000^2$, $\delta B/B \sim 3$, $\sigma \sim 1$

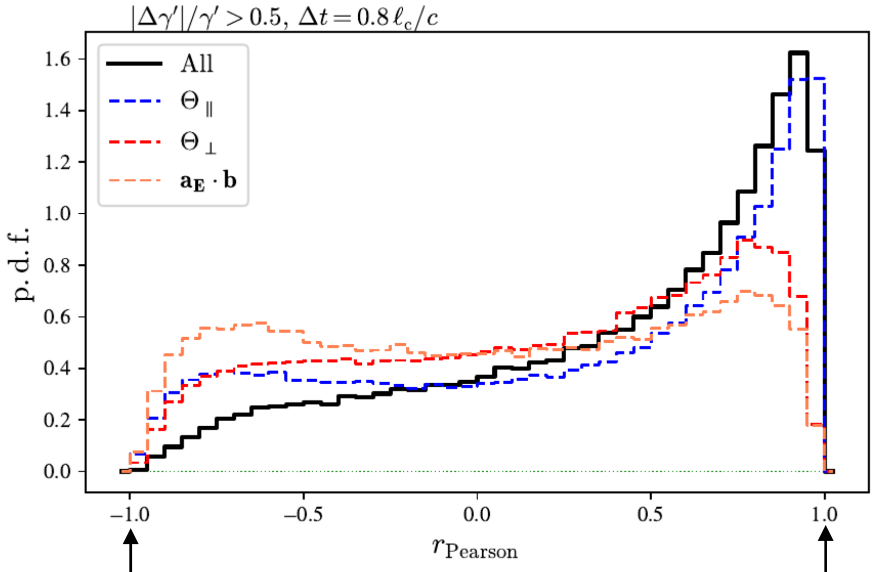
Comparison between model and simulations

→ model:

$$\frac{\mathrm{d}\gamma'}{\mathrm{d}\tau} = -\gamma' u'_{\parallel}\, \boldsymbol{a_E}\cdot\boldsymbol{b} - {u'_{\parallel}}^2\,\Theta_{\parallel} - \frac{1}{2}{u'_{\perp}}^2\Theta_{\perp}$$

→ comparison: for each particle history in a simulation, reconstruct γ’(t) using above model and velocity gradients measured in the simulation at **x**, t, then measure degree of correlation r_{Pearson} between the observed and reconstructed γ’(t)

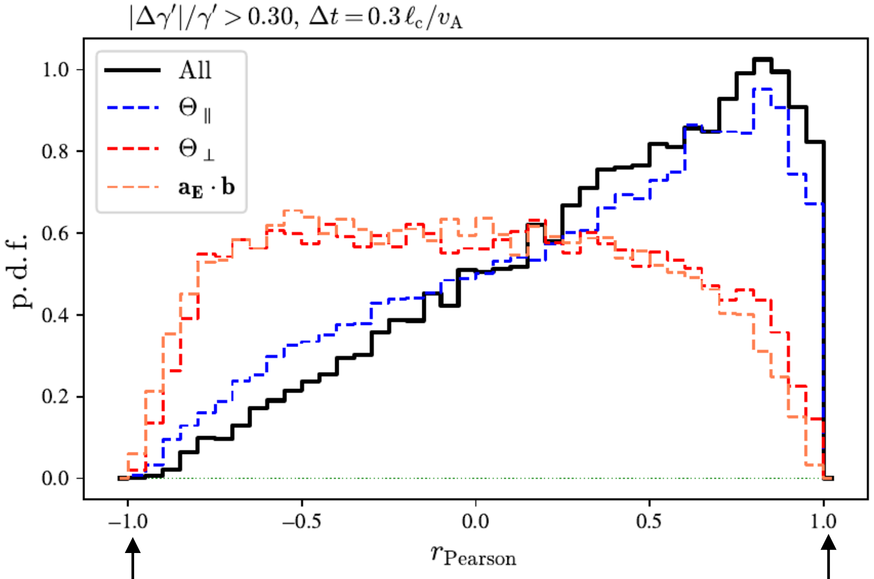
PIC simulation: 2D, 10 000^2, e$^-$e$^+$, $\delta B/B \sim 3$, $\sigma \sim 1$



100% anti-correlation

100% correlation

JHU driven incompressible MHD[1], 3D, 1024^3, $v_A = 0.4c$



100% anti-correlation

100% correlation

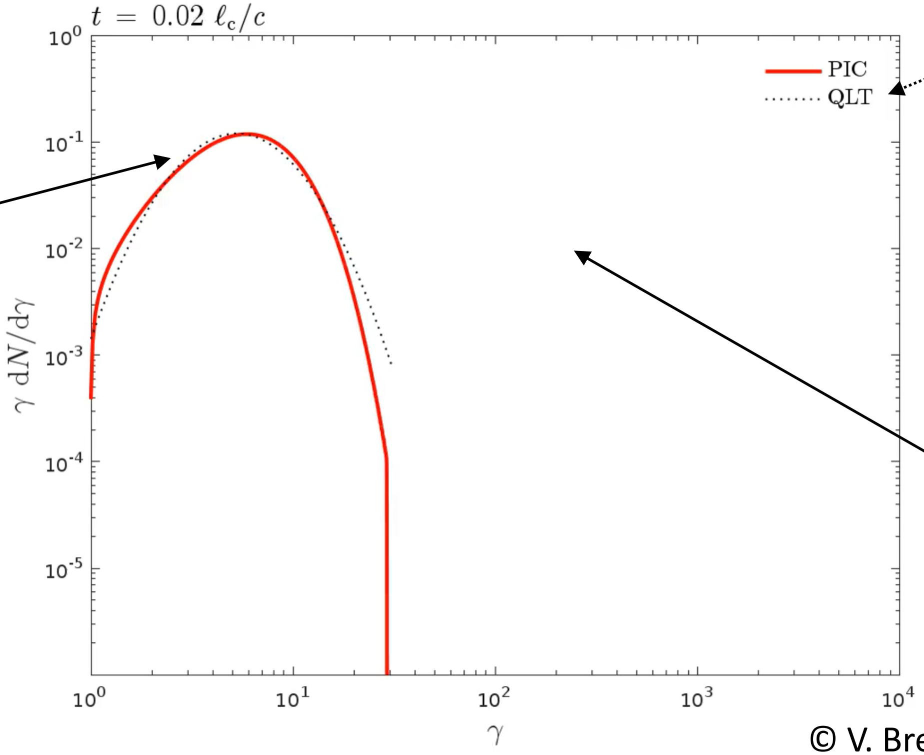
⇒ model captures the dominant contribution to particle energization
+ dominance of parallel shear contribution (field line curvature)

Refs.: 1. Johns Hopkins U. database, Eyink+ 13

Mismatch between PIC simulations and Fokker-Planck models...

→ Recent PIC simulations[1] reveal *nonthermal powerlaw spectra* (← Fermi acceleration in a closed box?!)

injection in thermal core:
from dissipation at kinetic (microscopic) scales, mostly reconnection



QLT/Fokker-Planck prediction:

FP: $$\frac{\partial}{\partial t} f(p,t) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\, D_{pp}\,\frac{\partial}{\partial p} f(p,t)\right]$$

⇒ sol.: $$p^3 f(p,t) \propto \exp\left[-\frac{\left(\ln p - \frac{3}{2}t/t_{\rm acc}\right)^2}{2t/t_{\rm acc}}\right]$$

$$t_{\rm acc} = p^2/D_{pp}$$

non-thermal tail:
from Fermi-type mechanism (no parallel E field)

© V. Bresci, L. Gremillet, M. L.: 2D PIC, driven, e^+e^-, 10 000^2, $\delta B/B \sim 3$, $\sigma \sim 1$

→ consequence: Fokker-Planck is not a good model... a powerlaw tail develops, drift is slow, unlike predictions!

→ Interpretation[2]: segregation in $t_{\rm acc}$ among particle population...

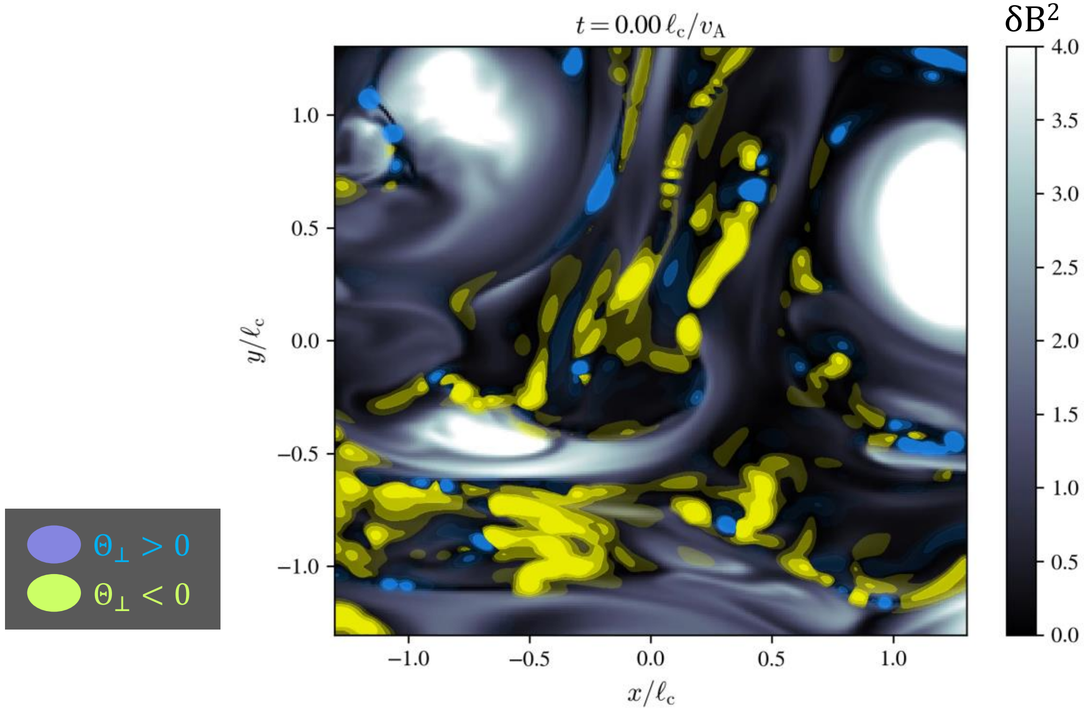
Refs: 1. Zhdankin+17-19, Wong+19, Comisso+Sironi 18,19; Nättilä+Beloborodov21 2. M.L. & Malkov 20

Acceleration in gradients: intermittency steps in

→ an important effect: strength of gradients grow on small scales…
p.d.f. non-Gaussian, *controlled by intermittency …*

⇒ localized (sparse) regions of intense gradients, with large powerlaw excursions…

⇒ mean free path to interaction can be macroscopic!



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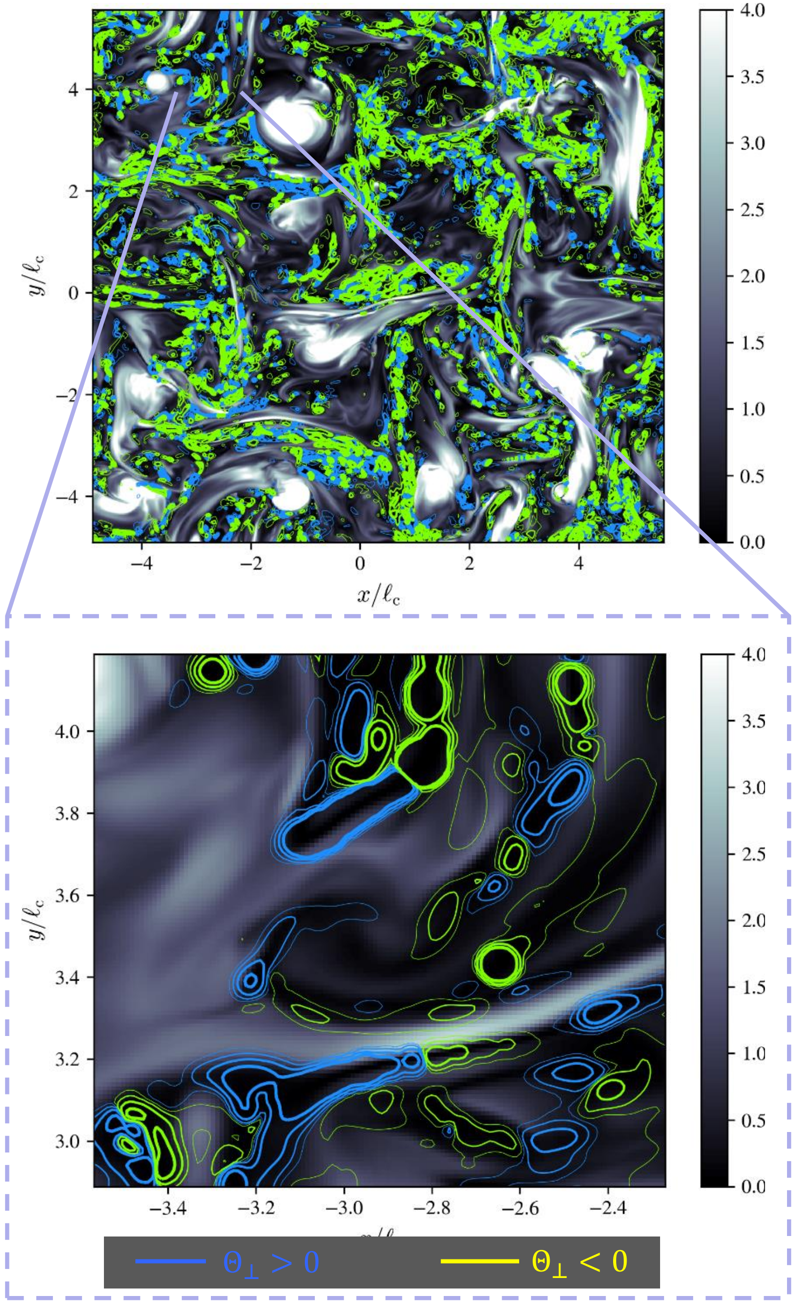
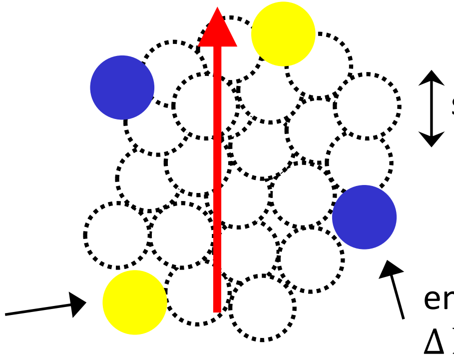


Non-trivial particle transport in intermittent, random velocity flows

→ an analytical (simplified) model for the spectrum:



step length l

energy gain
$\Delta \ln p = +g$

energy loss
$\Delta \ln p = -g$

on intermediate time scales t

… random walk with a small filling fraction f_+, f_-, of active sites…

… large momenta = ``lucky’’ particles

⇒ average rate of energy gain related to energy injection in plasma $(f_+ - f_-)$

⇒ presence of inactive regions implies existence of a powerlaw tail at large momenta, at all times…

… mean interaction time with active eddies:

$$t_{int} = l/(f_+ + f_-)c$$

Refs.: 1. e.g Isliker+17; Sioulas+20; ML+Malkov 20; ML21

Consequences of intermittency

→ analytical spectrum, main properties:

1. **one diffusion coefficient cannot capture the spectrum:**
diffusion coefficient

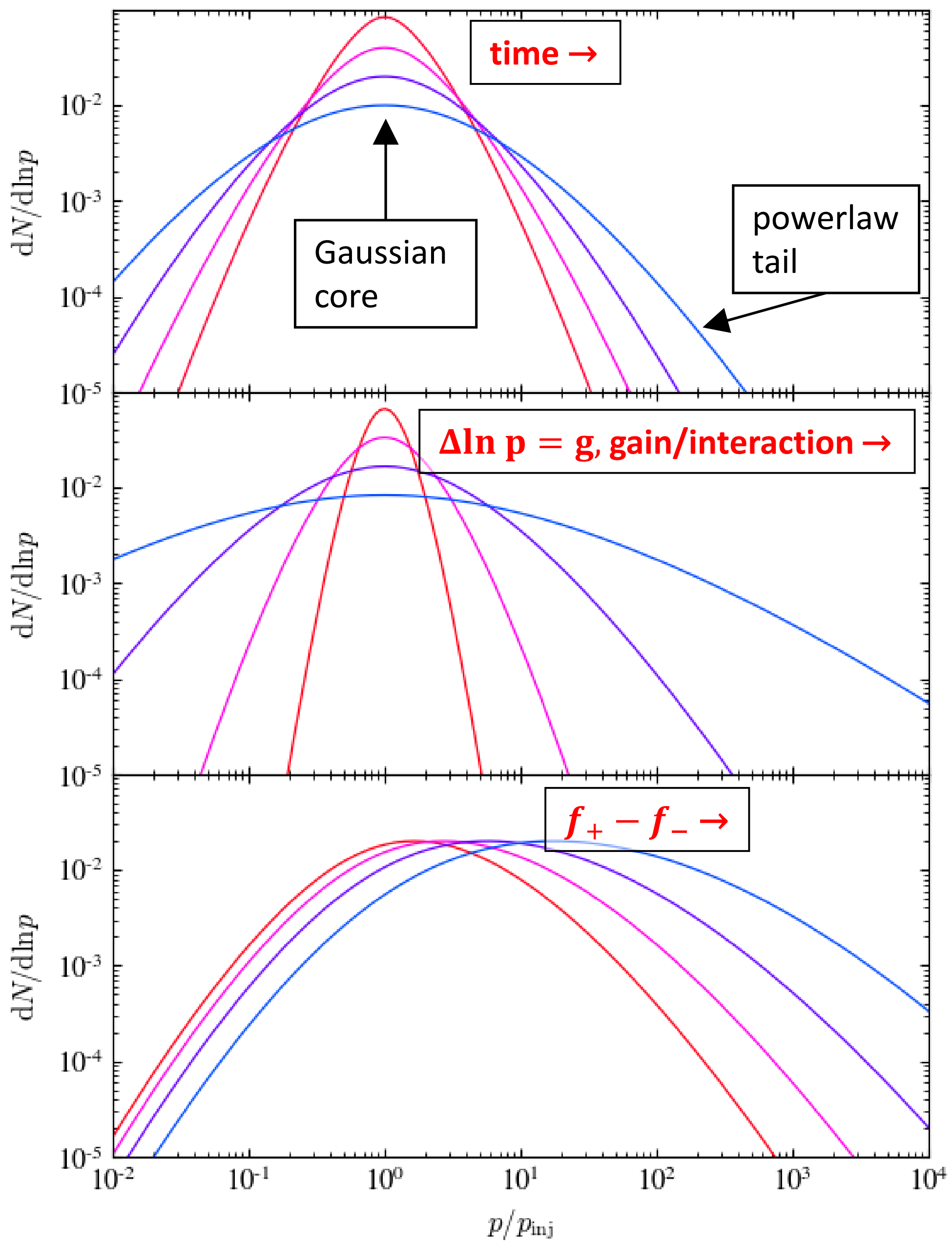
$$\frac{\langle \Delta p^2\rangle}{2t} = D_{pp} = \frac{p^2}{t_{\rm int}/g^2} \propto u_A^2\,\frac{p^2}{\ell_{\rm c}/c}$$

describes broadening of (thermal) Gaussian core, not powerlaw tail.

2. **a (quasi-)powerlaw tail subsists at all times, which hardens with time, and with increasing gain/interaction**
... as in PIC sims

3. **drift is slow, related to energy injection in the plasma**
... as in PIC sims
(vs: QLT assumes infinite reservoir of energy for particles!)

variations vs model parameters



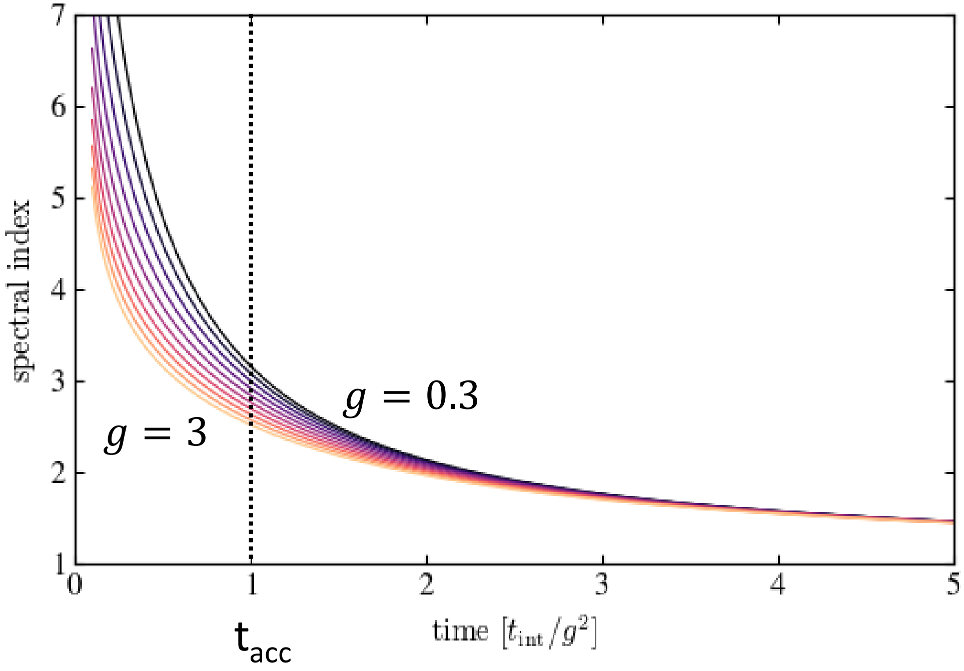
Spectral index as a function of time

→ main properties:

3. **acceleration timescale**: $\mathrm{t}_{\mathrm{acc}} \sim \frac{t_{\mathrm{int}}}{g^2} \sim c\ l_c/u_A^2$

... an average over the population: in reality, a distribution of acceleration timescales

4. **spectral index: ~ 3 for relativistic turbulence,**
softer for sub-relativistic…
steep spectra are generic (early on)



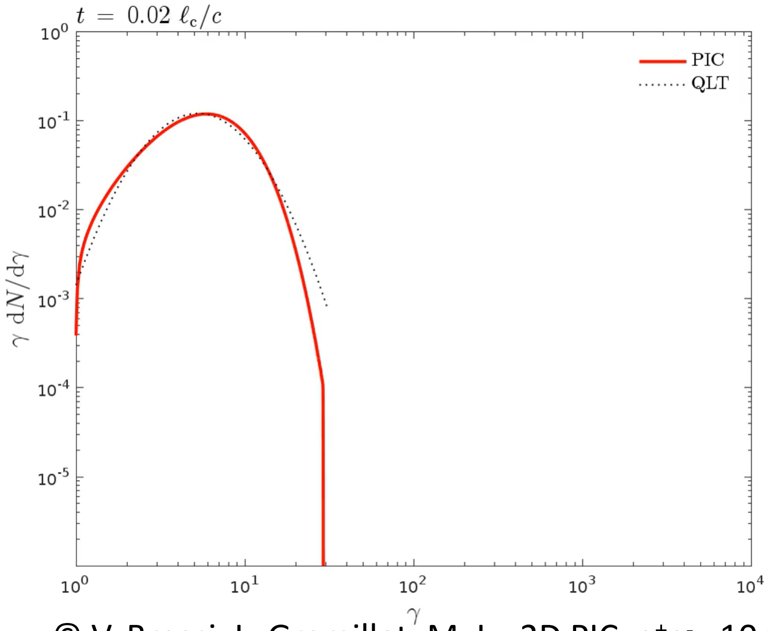
→ PIC simulations:

… acceleration timescale $\sim c\ l_c/u_A^2$

… at $u_A \sim 1$, observe index $\sim\ 3$…

… but, spectrum evolves very slowly beyond $\sim 10\ \ell_c/c$ as max momentum reaches model limit where $r_L \sim \ell_c$…

⇒ (near-)powerlaw observed at all times



© V. Bresci, L. Gremillet, M. L.: 2D PIC, e^+e^-, $10\ 000^2$, $\delta B/B \sim 3$, $\sigma \sim 1$

Summary

→ Stochastic particle acceleration in turbulent / random velocity flows:

1. particles gain energy non-resonantly in the large-scale (>r_L) sheared + compressive velocity flows

2. main sources of energy gain: [for isotropic scattering, shear and compression of u_E]
at $r_L \ll \ell_c$, shear along and compression transverse to *B*

3. acceleration regions strongly intermittent: sparse, localized in regions of gradients of B energy, with large powerlaw excursions in strength

4. general agreement of the model with PIC simulations:

⇀ fair reconstruction of particle histories (in momentum)
⇀ analytical random walk model reproduces the general trend of spectrum

5. *(near)* powerlaw spectra of accelerated particles appear generic

Some consequences for phenomenology and open questions

1. spectrum differs noticeably from std Fokker-Planck predictions

→ no pile-up distribution, quasi-powerlaw, slow drift: impact on phenomenology?
→ w/ improved model, including effects of radiative losses → recipe for inclusion in MHD/GRMHD simulations?

2. extrapolation to large hierarchy $\ell_c/(c/\omega_p)$… and other physical conditions

→ quasi-powerlaw (log-running), hardening in time vs PIC sims limited in dynamic range…
→ dependence on magnetization, beta-parameter, physics of stirring, composition etc.

3. impact of intermittency on transport, acceleration and radiative spectra

→ first experimental indication of ``anomalous'' transport: distribution of acceleration/scattering timescales ⇒?
→ on timescale ℓ_c/c, only a small fraction of particles has scattered ⇒ expect anisotropies on ℓ_c scales!
→ inhomogeneous particle spectra in one volume ℓ_c^3… consequences for flaring? (time profile?)
→ inhomogeneous spectra, u_E and B in one volume ℓ_c^3… consequences for radiative spectra?

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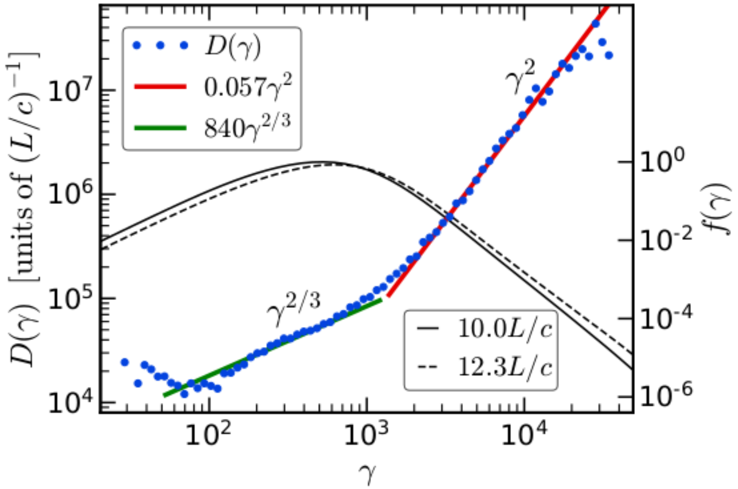
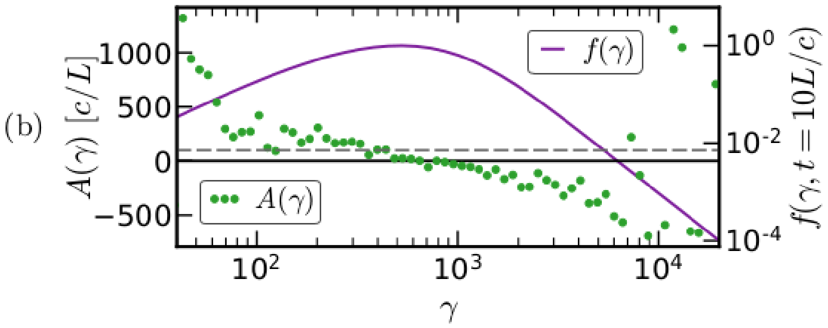
→ no pile-up distribution, quasi-powerlaw, slow drift: impact on phenomenology?
→ w/ improved model, including effects of radiative losses → recipe for inclusion in MHD/GRMHD simulations?

→ microphysical picture based on random walk: probability of energy gain based on $u_E(\boldsymbol{x}, t)$

→ alternative, Wong+19: extended FP equation with PIC-adjusted transport coefficients $D_{\gamma\gamma}$, A_γ

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \gamma}\left(D_{\gamma\gamma}\frac{\partial f}{\partial \gamma}\right) - \frac{\partial}{\partial \gamma}\left(A_\gamma f\right)$$





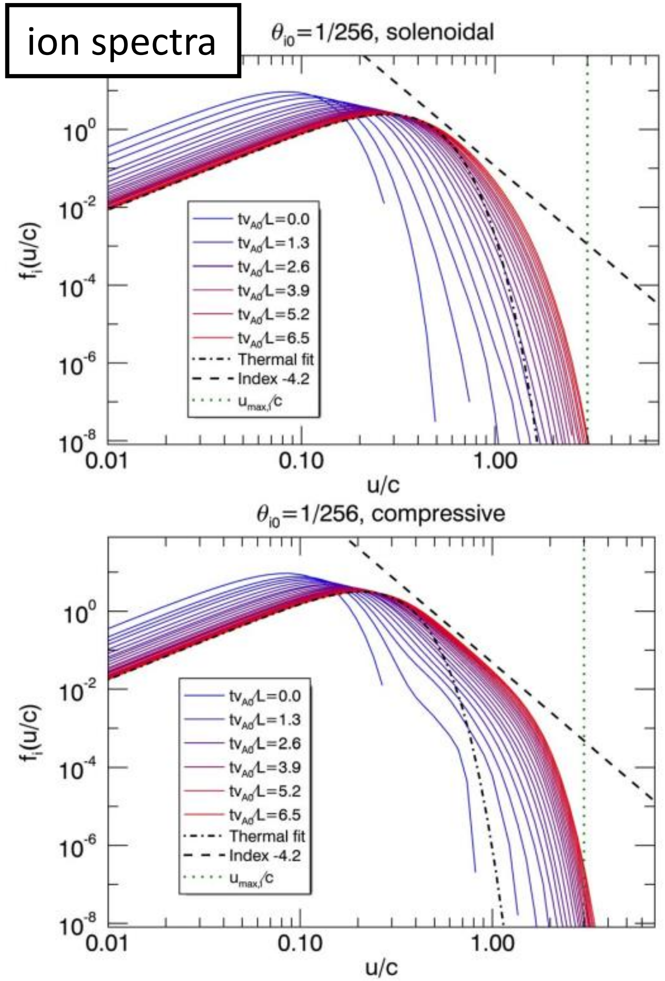
Some consequences for phenomenology and open questions

2. extrapolation to large hierarchy $\ell_c/(c/\omega_p)$… and other physical conditions

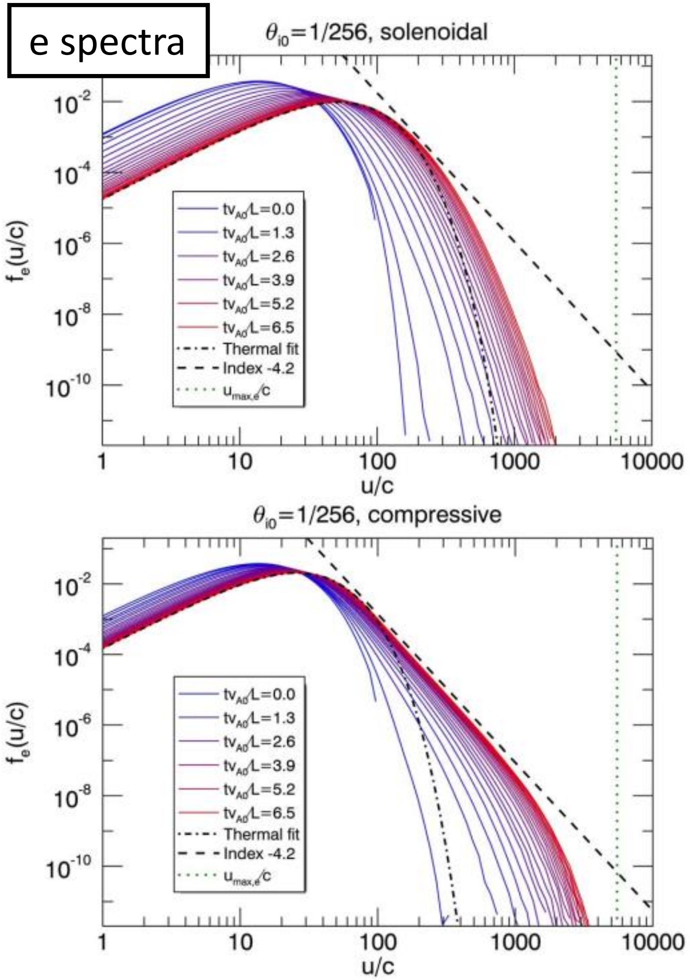
→ quasi-powerlaw (log-running), hardening in time vs PIC sims limited in dynamic range…
→ dependence on magnetization, beta-parameter, physics of stirring, composition etc.

Zhdankin 21:
e-ion PIC simulations at $\sigma = 0.02$,
Alfvén-like stirring vs compressive

ion spectra



e spectra

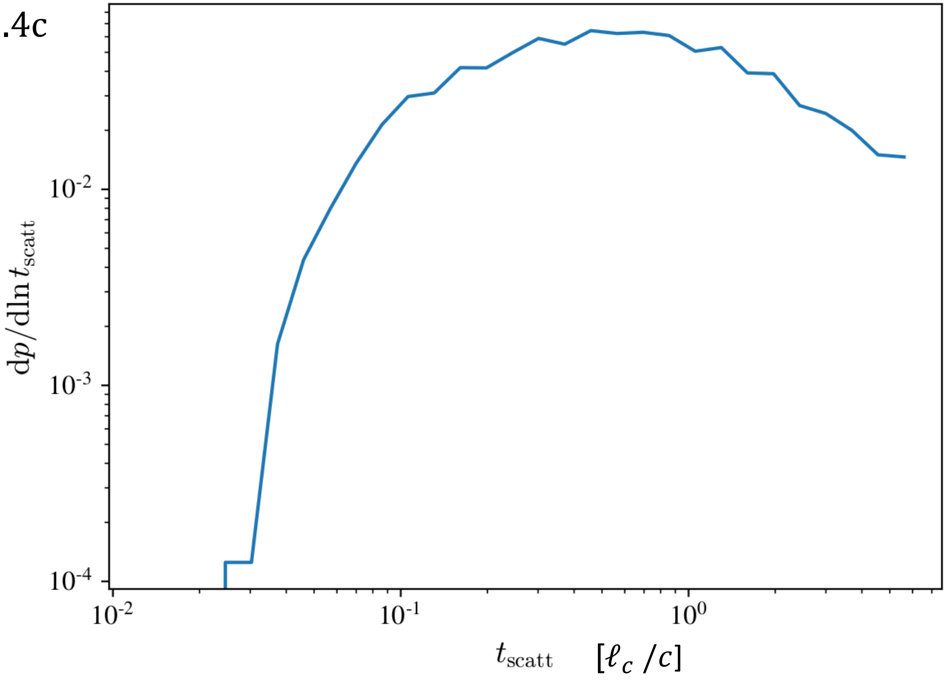


Some consequences for phenomenology and open questions

3. impact of intermittency on transport, acceleration and radiative spectra

→ first experimental indication of ``anomalous'' transport: distribution of acceleration/scattering timescales ⇒?
→ on timescale ℓ_c/c, only a small fraction of particles has scattered ⇒ expect anisotropies on ℓ_c scales!
→ inhomogeneous particle spectra in one volume ℓ_c^3… consequences for flaring? (time profile?)
→ inhomogeneous spectra, u_E and B in one volume ℓ_c^3… consequences for radiative spectra?

JHU incompressible MHD, 3D, 1024^3, $\mathrm{v_A} = 0.4c$



distribution of scattering timescales: expect strong anisotropies on ℓ_c length scales!

Some consequences for phenomenology and open questions

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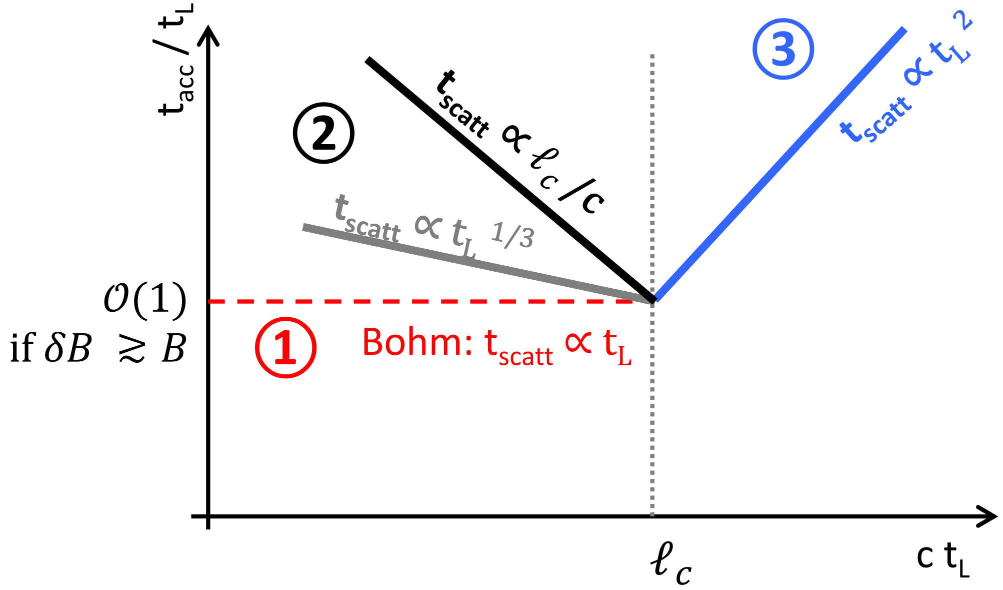

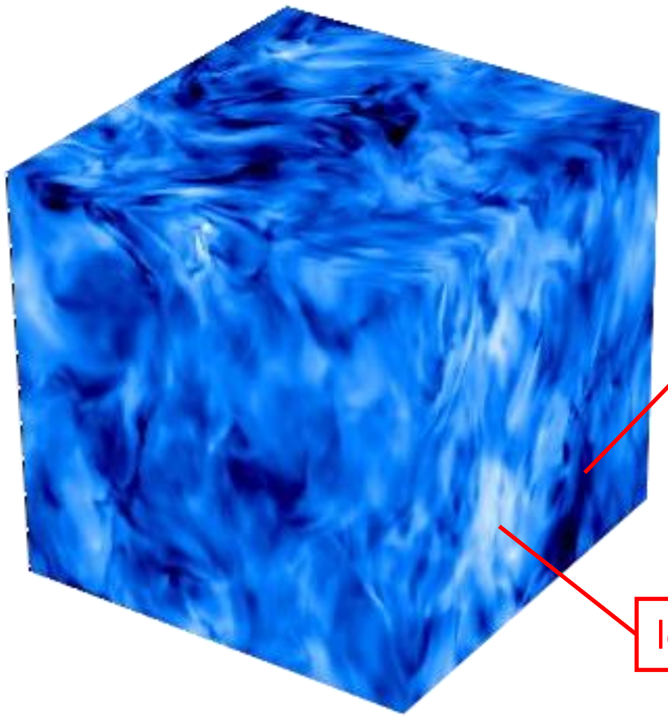

local spectrum = boost(f, u_E, B)

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e.g., Bykov+13 in connection to Crab flares, Khangulyan+21 for synchrotron in inhomogeneous B

Some consequences for phenomenology and open questions

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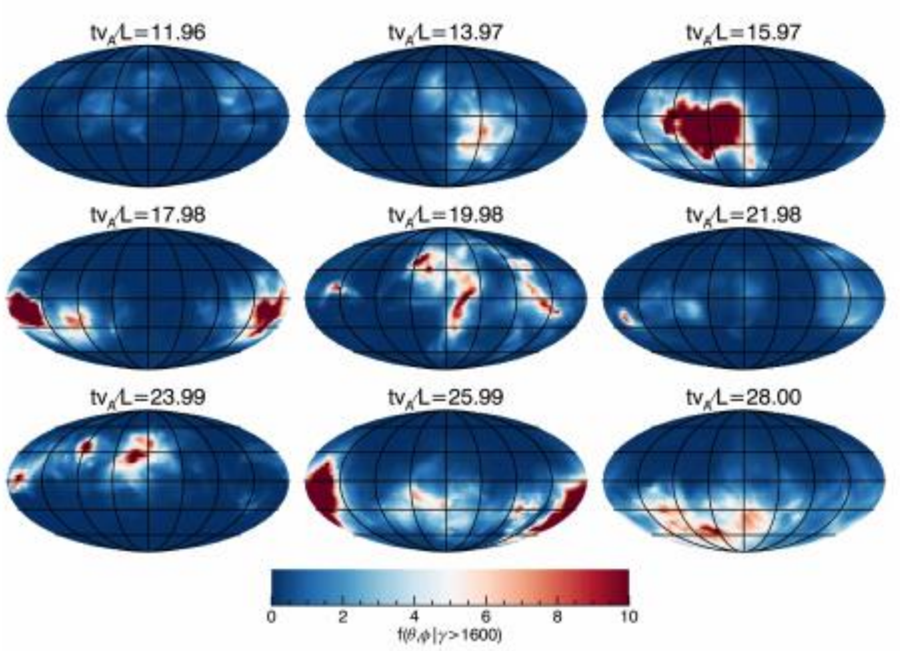
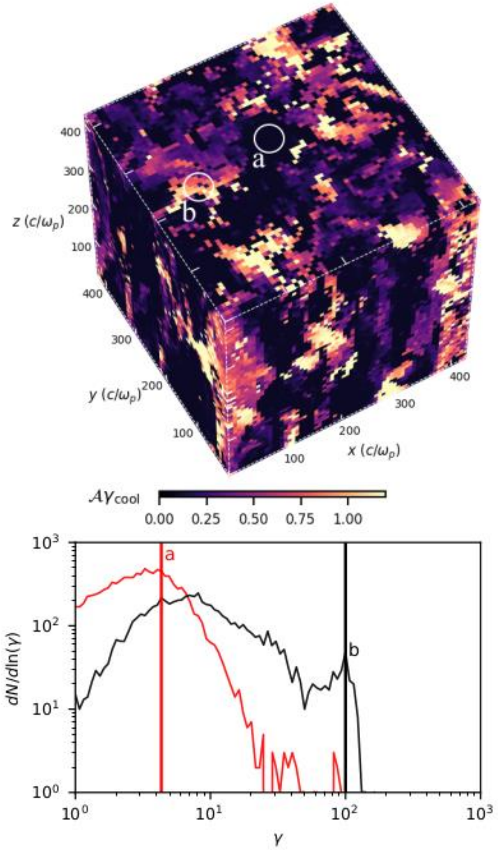
Zhdankin+18:
PIC, relativistic + radiative sims,



anisotropic momentum distribution at large momenta

Nättilä+Beloborodov 20:
PIC, relativistic + radiative sims,



Some consequences for phenomenology and open questions

1. spectrum differs noticeably from std Fokker-Planck predictions

→ no pile-up distribution, quasi-powerlaw, slow drift (connected to energy injection): impact on phenomenology?
→ w/ improved model, including effects of radiative losses → recipe for inclusion in MHD/GRMHD simulations?

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→ quasi-powerlaw (log-running), hardening in time vs PIC sims limited in dynamic range…
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